Tasks, Occupations, and Wage Inequality in an Open Economy

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Abstract

This paper documents and theoretically explains a nexus between globalization and residual wage inequality through internal labor market reorganization. Combining time-varying within-occupation task information from representative German labor force surveys with linked plant–worker data for Germany, we establish three interrelated facts: (1) Larger plants and exporters organize production into more occupations, and (2) workers at larger plants and exporters perform fewer tasks within occupations, while (3) overall and residual wages are more dispersed at larger plants. To explain these facts, we build a model in which the plant endogenously bundles tasks into occupations and workers match to occupations. By splitting the task range into more occupations, the plant can assign workers to a narrower task range per occupation, reducing worker mismatch and raising worker efficiency as well as the within-plant dispersion of wages. Embedding this rationale into a Melitz (Econometrica 2003) model, where fixed span-of-control costs increase with occupation counts, we show that inherently more productive (exporter) plants exhibit higher worker efficiency and wider wage dispersion and that economy-wide wage inequality is higher in the open economy for an empirically confirmed parametrization. Estimation of our model shows that a worker’s average number of tasks is inversely related to plant revenues and that the within-plant wage dispersion is positively related to plant size.

JEL-Classification: F12, F16, L23

Keywords: International Trade, Firm-Internal Labor Allocation, Heterogeneity

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“It is the great multiplication of the productions of all the different arts, in consequence of the division of labour, which occasions, in a well-governed society, that universal opulence which extends itself to the lowest ranks of the people.”
— Adam Smith (1776): The Wealth Of Nations, Book I, Chapter I

1 Introduction

Recent theories of international trade at the firm level have opened new insights into a nexus between globalization and wage inequality within sectors and occupations. Much of the emphasis to date has been on the wage dispersion between firms, given the wage premia that exporters pay to otherwise similar workers within sectors and occupations (Helpman, Itskhoki and Redding 2010, Egger and Kreickemeier 2009, Davis and Harrigan 2011, Amiti and Davis 2012). The empirical importance of the between-firm or between-plant dispersion of wages for changes in overall wage inequality has been documented for labor markets in general (Card, Heining and Kline 2013, Lopes de Melo 2013, Song et al. 2015) and for labor market outcomes in open economies in particular (Egger, Egger and Kreickemeier 2013, Coşar, Güner and Tybout 2015, Helpman et al. 2017, Eaton, Kortum and Kramarz 2015). In the cross section of workers, however, the commanding component of wage variation is within firms: studies such as Abowd et al. (2001) and Menezes-Filho, Muendler and Ramey (2008), for instance, control for worker and employer characteristics, as well as firm effects, in Mincer regressions and show a dominance of the residual wage component; Lemieux (2006) documents the response of the residual wage component to economic change. In this paper we relate back to the basic principle of the division of labor, within plants and within occupations across workers as well as across countries in the global economy. We explore how the large within-plant part of wage inequality responds to trade—through internal labor-market reorganization—and show that the size of a plant’s global product market translates into its internal division of labor, so that global specialization affects inequality across the “ranks of the people.”

In his foundational analysis of the division of labor, Adam Smith (1776, Book I, Chapter I) described tasks that a single worker could perform cumulatively or that the employer could alternatively assign to several workers:

“[M]aking a pin is . . . divided into about eighteen distinct operations. . . . [T]en persons . . . could make among them upwards of forty-eight thousand pins in a day. But if they had all wrought separately and independently . . . they certainly could not each of them have made twenty, perhaps not one pin in a day.”

To elicit information on cumulative tasks in Adam Smith’s operational sense, and on the organization of the workplace in today’s economy, we use the six German Qualifications and Career Surveys (BIBB-
BAuA surveys) conducted over the years 1979 through 2012 in six-to-eight-year intervals and build time-consistent measures of workplace activities. The survey data allow us to discern 15 cumulative workplace activities that are consistently reported over the three decades (examples are: Produce Goods; Develop, Research, Construct; Organize, Plan, Prepare; and Oversee, Control Machinery or Processes). Importantly for an understanding of the evolving division of labor between employers, the BIBB-BAuA surveys also allow us to quantify how many tasks workers perform over time and across plants but within their occupations (jobs). We combine the task information by occupation, industry, location and plant size with German linked plant–worker data (IAB-LIAB). The employer data comes at the plant level and from annual surveys, which are available since the 1990s. We therefore mostly use the four BIBB-BAuA waves since 1992 to map task information at the worker level to LIAB.

Three striking facts emerge. First, larger plants and plants that are predicted to become exporters adopt a wider count of occupations. Second, however, workers at these larger plants perform, on average, narrower ranges of tasks within occupations. In other words, increasing plant-level trade openness promotes the internal division of labor. Third, both overall and residual wages are more dispersed within larger plants, conditional on the occupations of workers. Our hypothesis is that workers differ in their unobserved ability to conduct the tasks in an occupation, so that mismatches result in varying labor efficiencies within an occupation. To the extent that wages are linked to worker efficiency, the ability mismatches generate wage inequality—in accordance with the empirical observation that the major part of residual wage inequality (that is wage dispersion not explained by observable differences of workers) materializes within plants, within layers of hierarchy, and within occupations.

To explain these facts, we propose a model of endogenous occupation choice and task assignments by the employer. Employers can organize the full range of tasks that need to be performed for production into fewer or into more occupations (jobs). A smaller count of occupations at a plant implies that the workers who do the jobs have to carry out a wider range of tasks. Conversely, in plants with a larger count of occupations, each job only requires a narrower range of tasks to be performed. In other words, the finer the task space is divided into jobs, the more specialized in fewer tasks a worker in each job can become. We postulate that workers have a core ability that makes them most efficient at one particular task in the full task range and monotonically less efficient at tasks that are more distant from their core

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1We combine all six BIBB-BAuA surveys for the first time in this paper, constructing time-consistent task measures. Select survey questions and years have been used in earlier research, for example by Acemoglu and Pischke (1998), Spitz-Oener (2006), Gathmann and Schönberg (2010) and Becker and Muendler (2015).

2The BIBB-BAuA surveys also report descriptive task aspects, such as whether work is intensive in cognitive or routine work steps. Those task properties are not necessarily cumulative in Adam Smith’s operational sense (see e.g. Becker and Muendler 2015, documenting a roughly constant frequency of those task aspects over the period 1979-2006).
ability. Workers assortatively match to task ranges that include their core ability. As a consequence, when a plant’s task ranges are narrower, then the degree of mismatch between a worker and the tasks is smaller because all of an occupation’s tasks are closer to a worker’s core ability. Workers are therefore more efficient at plants with more occupations and a finer division of labor.

In such a world, all plants would opt to divide the task range into possibly many jobs if this division were costless. To explain finite occupation counts at plants, and the observed larger count of occupations at larger plants, we posit that there is a plant-level fixed cost of operation that increases with the count of occupations. Managing additional occupations and coordinating workers across more specifically defined jobs causes a span of control problem whose cost we assume to increase in the count of occupations. As is well established in Melitz (2003) models with heterogeneous producers, more productive plants recover the fixed costs with their operating profits, and in our framework this implies that the equilibrium outcome results in the adoption of a larger count of occupations at plants that are inherently more productive. In particular, relatively productive plants that also select into exporting will choose more occupations with narrower task ranges compared to non-exporters.

The model has a number of testable features and predictions. Chief among the features is the inverse relationship between a plant’s count of occupations and the width of its average task range per occupation. We document that this inverse relationship between occupation counts and task ranges robustly holds, using both linear predictions and instrumental-variable approaches in the spirit of Autor, Dorn and Hanson (2013) and Dauth, Findeisen and Suedekum (2014) by instrumenting endogenous plant-level regressors, such as (export) sales, the occupation counts, and interactions of those two, with foreign market shocks from China and Eastern Europe. The model also offers concrete predictions for the within-plant wage inequality across workers—a dominant component of wage inequality. If a basic magnification parameter is positive, and thus adds to worker efficiency from specialization on a narrow task range, then more productive, larger plants exhibit higher within-plant wage inequality than less productive, smaller plants. We document in this paper for both overall wages and the residual wage component in German manufacturing data that the within-plant wage dispersion is higher at more productive plants. Instead of considering total wages or wage residuals that exclude the part predicted by worker characteristics, as we do here, Bombardini, Orefice and Tito (2015) focus on a permanent worker-specific and time invariant wage component, measured alternatively with average observed lifetime earnings or with a worker fixed effect following Abowd, Creecy and Kramarz (2002). They find that the long-term wage component is less dispersed at larger, more productive French manufacturing firms. In light of our model, the use of
a permanent worker-specific and time invariant wage component to proxy worker efficiency is akin to working with a negative magnification parameter.

The relationship between globalization, technical change and earnings inequality is arguably a crucial concern for policy and economic analysis. Our model lends itself to log-linear relationships that can be estimated structurally, using maximum likelihood and GMM approaches. In this draft, we state the estimable relationship but have to defer the inclusion of structural estimation results, and policy simulations on their basis, to future versions of this paper. Some relationships are directly estimable with individual linear regressions, however. In particular, we find that the elasticity of operational fixed costs with respect to the count of occupations is significant, though not excessively large in magnitude, suggesting that employers have scope to raise occupation counts and narrow task ranges per occupation. We also find that the estimated magnification parameter, which regulates the relationship between plant size and within-plant wage dispersion in the model, is strictly positive for both overall wages and residual wages.\(^3\) This result reconfirms, in the light of our model, our reduced-form regression results that more productive plants exhibit higher within-plant wage inequality.

The positive estimate for the model’s magnification parameter has direct implications for welfare and wage inequality. In the model, opening up to trade leads to a selection of the most productive plants into exporting, raising welfare, but results in an asymmetric response in plant-level wage inequality. The variance of wages increases in exporting plants, if wage inequality was already high at these producers under autarky, while within-plant wage inequality declines at non-exporters. Given the asymmetry in plant-level implications, access to foreign trade exerts counteracting effects on economy-wide wage inequality. We can show, however, that economy-wide wage inequality is higher in the open than in the closed economy if and only if the magnification parameter is positive. Our preliminary positive estimate for the magnification parameter therefore suggests that globalization aggravates economy-wide wage inequality in all open economies around the world.

Workplace tasks are an important, employer-driven characteristic of the labor market, and have been documented to relate closely to recent labor market changes including wage polarization (Autor, Katz and Kearney 2006, Goos, Manning and Salomons 2009) and the offshorability of jobs (Leamer and Storper 2001, Levy and Murnane 2004, Blinder 2006). The assignment of tasks in an open economy, and the implications for welfare and wage inequality, have been studied from a theoretical perspective in industry-level models, including the Heckscher-Ohlin (Grossman and Rossi-Hansberg 2008, 2010) and

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\(^3\)The lack of full longitudinal worker data in the IAB-LIAB random sample of plants does not allow us to replicate the permanent wage component measures from Bombardini, Orefice and Tito (2015) in our context.
the Ricardian framework (Rodríguez-Clare 2010, Acemoglu and Autor 2011). Our model complements the industry-level perspective with a plant-level view. Beyond considerations of offshorability, our treatment of tasks emphasizes the quality of the worker-task match as a key determinant of plant performance and in this regard relates closely to studies of internal labor markets by Barron, Black and Loewenstein (1989), who show that the quality of worker-task matches reduces on-the-job training costs, as well as Meyer (1994) and Burgess et al. (2010), who document that the quality of worker-task matches raises team efficiency and the effectiveness of incentives.

Human resource management practices have been found to be an important determinant of the variation in plant and firm productivity within and across countries (Bloom and Van Reenen 2011). Yet, aspects of the internal labor market and residual wage inequality are difficult to observe directly. Recent studies of the firm’s internal labor market have turned to the importance of observable hierarchies (Caliendo and Rossi-Hansberg 2012, Caliendo, Monte and Rossi-Hansberg 2015) and their response to firm-level trade. Our model complements the hierarchical approach to a firm’s internal organization with a perspective on the horizontal differentiation of worker abilities and their tasks within hierarchical layers. In fact, we find that most employer-level residual wage inequality in the German data is also within hierarchies (and within occupation categories), suggesting that an important horizontal wage differentiation component acts within hierarchies. The internal organization of plants and firms also involves the motivation of workers to exert effort. Related studies analyze the response of employers’ incentives for workers, and observable incentive pay in particular, when global competition changes (Guadalupe 2007, Cunat and Guadalupe 2009). Our paper complements the view on incentives for worker effort with a perspective on management responses to product-market opportunities, as employers adjust the observable count of occupations they offer and coordinate the observable range of tasks they assign within jobs.

An alternative approach to modelling worker-level wage dispersion within and between employers considers the employer-worker matching process (see e.g. Legros and Newman 2002, Eeckhout and Kircher 2011). The potential efficiency gains from improved assortative matching have received according attention in the trade literature (Costinot and Vogel 2010, Sampson 2014). Several studies highlight trade-induced changes in match quality as a key aspect of trade in terms of welfare, employment and wage inequality (Amiti and Pissarides 2005, Davidson, Matusz and Shevchenko 2008, Davidson et al. 2014). More recent studies have started to complement the analysis of cross-industry and cross-firm matches

4The literature estimating search models of the labor market more generally includes Burdett and Mortensen (1998), Cahuc, Postel-Vinay and Robin (2006), Postel-Vinay and Robin (2002), and Postel-Vinay and Turon (2010).
with an analysis of within-firm matches. Larch and Lechthaler (2011) study the assignment of workers across plants within multinational firms, and Bombardini, Orefice and Tito (2015) investigate the permissible ability ranges of workers at firms when worker-firm matches are formed. Our model highlights that an additional source of efficiency gains for employers is to improve match quality by narrowly assigning tasks to workers with the best fit to those tasks (a core ability within the occupation’s task range). Our worker-reported task frequencies within occupations characterize empirically the assignment of workplace activities to workers at different plants.

In our model relatively more productive plants choose to augment their elemental productivity with narrower task ranges that make their workers more efficient, thus concentrating the firm size distribution beyond the inherent productivity dispersion and introducing a feedback effect on the market entry and export participation decisions. While the principal selection of more productive firms into exporting remains a basic force in the model (as documented empirically by Clerides, Lach and Tybout 1998, e.g., and others), the feedback of exporting into worker efficiency through narrower task ranges at exporters is akin to a learning-by-exporting effect (for direct evidence on learning-by-exporting see, e.g., Crespi, Criscuolo and Haskel 2008). The labor market feedback effect in our model is similar to the outcome of screening in Helpman, Itskhoki and Redding (2010) and the effect of investment into innovations in Aw, Roberts and Xu (2011). In the Helpman, Itskhoki and Redding (2010) model, screening for higher ability workers raises the returns to exporting and vice versa; in the Aw, Roberts and Xu (2011) model, R&D investments raise the returns to exporting and vice versa; in our model, improving the worker-task match quality raises the returns to exporting and vice versa.

The remainder of this paper proceeds as follows. In Section 2, we present our data, explain the construction of time-varying task measures and their combination with longitudinal plant data, and collect descriptive evidence that motivates our model. In Section 3, we build a model of production with task assignment to occupations and derive the equilibrium for a closed economy. We extend the model to open economies that trade final goods in section 4. In Section 5 we estimate key relationships of the model and subject it to empirical tests. There, we also provide preliminary evidence in support of the formal structure of our model but defer a detailed structural estimation of our model to future versions of our manuscript. Section 6 concludes.

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5 In a review of the literature on the structure of wages within and across firms, Lazear and Shaw (2009) conclude that the wage structure appears to be more dependent on firm- or within-plant sorting of workers to occupations than on sorting of workers to firms or plants.

6 An interpretation related to the core ability of workers, most suitable for specific tasks, is that human capital is occupation specific. Kambourov and Manovskii (2009) and Sullivan (2010) provide empirical evidence on occupation-specific human capital.
2 Data and Descriptives

The two main sources for our novel micro-level data on employer-level task assignments are (i) the German Qualifications and Career Surveys (BIBB-BAuA surveys), and (ii) the Linked Plant–Worker Data provided by IAB (LIAB). Additionally we use sector-level bilateral merchandise trade data from the United Nations Commodity Trade Statistics Database (Comtrade) and service trade from the trade in services database (TSD) from the World Bank. Importantly, we are able to consolidate the different sector level definitions and construct 39 longitudinally consistent industries for all data sources. Our consistent sector definition across all data sources is based on an aggregation of NACE 1.1 for the European Communities, which is equivalent to the German Klassifikation der Wirtschaftszweige WZ 2003 at the 2-digit level (see Becker and Muendler 2015).

2.1 Labor force survey data

Observable information on the organization of the workplace is taken from six German Qualifications and Career Surveys conducted over the years 1979 through 2012 by Germany’s Federal Institute for Vocational Education and Training BIBB (most recently in collaboration with with think tank BAuA). Each wave is based on a sampling frame that selects a random sample of around one-tenth of a percent of the German labor force with more than 20 hours of work during the survey week. The BIBB-BAuA data reports detailed information on workplace properties, worker characteristics, the industry, occupation and earnings, as well as some rudimentary information on the employer, such as the size of a worker’s plant in seven size categories.7 Most importantly, we observe workers’ responses to survey questions that regard the tasks they perform on the job. Following the time consistent definitions in Becker and Muendler (2015), we append the 2012 survey data and make use of the questions that elicit what activity workers carry out on the job. A worker may report these activities as performed or not. We can discern 15 such activities, reported in a time consistent manner throughout all six BIBB-BAuA survey waves: 1. Manufacture, Produce Goods; 2. Repair, Maintain; 3. Entertain, Accommodate, Prepare Foods; 4. Transport, Store, Dispatch; 5. Measure, Inspect, Control Quality; 6. Gather Information, Develop, Research, Construct; 7. Purchase, Procure, Sell; 8. Program a Computer; 9. Apply Legal Knowledge; 10. Consult and Inform; 11. Train, Teach, Instruct, Educate; 12. Nurse, Look After, Cure; 13. Advertise, Promote, Conduct Marketing and PR; 14. Organize, Plan, Prepare (others’ work); 15. Oversee, Control Machinery and Technical Processes. These workplace activities (what tasks) are cumulative and exhibit

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7Plant size categories are 1-4; 5-9; 10-49; 50-99; 100-499; 500-999; 1000 or more workers.
Table 1: Simultaneous Activities By Survey Wave

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Total: 1.000 1.000 1.000 1.000 1.000 1.000
Average: 1.676 2.177 2.105 5.250 7.261 7.316
Observations: 29,737 26,361 24,090 27,634 16,964 16,718


a pronounced change towards more multitasking over time until the early 2000s, with a relatively stable level of multitasking from then on. As documented in Table 1, German workers perform on average six times more of the 15 possible activities in 2006 than they did in 1979.

Research into tasks and wage polarization (e.g. Autor, Katz and Kearney 2006, Goos, Manning and Salomons 2009) or tasks and offshorability (Leamer and Storper 2001, Levy and Murnane 2004, Blinder 2006, e.g.) frequently considers a different dimension of tasks, including the routineness of work steps and codifiability of job descriptions, which are also reported in the BIBB-BAuA surveys. Becker and Muendler (2015) call those tasks, which are related to how workers conduct their work, performance requirements and document that those tasks exhibit little time variation even though they are not mutually exclusive tasks. In contrast to the (what) activities above, German workers do not report more simultaneous performance requirements over time (Becker and Muendler 2015, Table 2). For the purposes of our model of a plant’s internal labor market, and its response to product-market shocks at home and abroad, we are most interested in tasks that are empirically found to be cumulated at the workplace into
multitasking jobs. We therefore restrict our attention to the 15 (what) activities in the BIBB-BAuA data.

2.2 Linked plant–worker data

We use data hosted at the German Federal Employment Office’s Institute for Employment Research (IAB): the Linked Plant–Worker Data at the IAB (LIAB). The LIAB data provides record linkages for matching detailed administrative data on workers registered with the German social security system to the IAB Plant Panel.\(^8\) The plant data covers detailed plant information from employer surveys on an annual basis since 1993. Information on plants located in the Eastern part of Germany is only available since 1996. We therefore restrict the sample period to the years 1996-2014 to cover the German economy as a whole. At the plant level we use information on revenues, export status and export revenues, employment together with region and sector categories. At the individual level we draw on demographic, tenure and education indicators, occupation characteristics, and data on daily wages.\(^9\) Larger plants are over-represented in the plant panel. We therefore use the weighting factors provided by IAB and make our plant-level data representative for the German economy as a whole, because we want our statistics to reflect economy-wide effects.

In linking individual worker data to plant data, the LIAB records allow us to quantify sources of wage variation in the labor market. To assess wage inequality, we first remove observed demographic, education and tenure information together with time, industry and region effects from log daily wages in a Mincer regression, and obtain residual log daily wages. We take those observed characteristics out because they are covered by classic trade theory. They explain about 53 percent of the log wage variation (42 percent if we omit industry effects). Similar to other studies for both industrialized and developing countries (see e.g. Abowd et al. 2001, Menezes-Filho, Muenler and Ramey 2008), this finding implies that almost half of the wage dispersion remains unexplained at this level of analysis. Table 2 follows up with further decompositions of the variance of the (exponentiated) log daily wage residual. Variation between plants explains about 24 percent of residual daily wages (exponentiated Mincer residuals) in 1996-2014 (column 2), still leaving 76 percent of residual wages unexplained. Looking at the variation between plants and between their managerial hierarchies brings the unexplained part of residual wage dispersion down by another 7 percentage points in 1996-2014 (to 69 percent). Occupations are perfectly nested within hierarchies (using the occupation-to-hierarchy mapping from Caliendo, Monte

\(^9\)Wage information in the social security records is right-censored, so we replace censored wages by imputed wages, following the procedure proposed by Baumgarten (2013).
Table 2: Decompositions of Residual Wage Inequality in Plant–Worker Records

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<td>52</td>
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Notes: Residual log daily wage from standard Mincer regression (exponentiated in four latter columns), taking out demographic, education and tenure information as well as time, industry and region effects ($R^2 = 53\%$). ($^* Mincer regression excludes industry effects, R^2 = 0.42.$) 357 occupations at the 3-digit KldB-88 level. The variance of the log hourly wage $w_{it}$ is linearly decomposed into a within and a between part. The reported percentages are the contribution of the within component to the total.

and Rossi-Hansberg 2015). Considering the residual daily wage variation between plants and between their occupations (357 occupations at the 3-digit KldB-88 level) pushes the unexplained part further down by another 11 percentage points. However, 58 percent of the residual daily wage variation remain unexplained even at the plant-occupation level. In other words, much wage variation (58 percent of the 47 percent not explained with Mincer regressions) occurs within plant-occupations. That is the wage variation we take on in this paper.

2.3 Trade data

Information on Germany’s sector-level imports and exports with China and Eastern Europe is taken from the United Nations Commodity Trade Statistics Database (Comtrade) and the trade in services database (TSD) at the World Bank. To construct instruments for German exports and imports we follow Autor, Dorn and Hanson (2013) and Dauth, Findeisen and Suedekum (2014) and use shipments between Australia, Canada, Japan, Norway, New Zealand, Sweden, Singapore, and the United Kingdom on the one hand and China and Eastern Europe on the other hand as the instrument group. We map the SITC Rev. 2 sector information to a common sector definition across all waves of the German data. To create a concordance from SITC Rev. 2 to the 39 longitudinally consistent industries, we rely on existing mappings from SITC Rev. 2 to ISIC Rev. 3.1 and from ISIC Rev. 3.1 to WZ 2003.

10Trade flows are converted into Euros using yearly exchange rates supplied by the German Bundesbank.
2.4 Data combination

A meaningful analysis of the within-plant-occupation component requires measurable properties of occupations. The BIBB-BAuA labor force survey offers such variables: consistently defined tasks within occupations, across employers and over time. As discussed in subsection 2.1 above, we can distinguish 15 different time-consistent activity-related (what) tasks in the BIBB-BAuA labor force surveys. Task information is not available in the linked plant–worker data LIAB. We therefore propose to combine the BIBB-BAuA worker survey information with the LIAB records through imputation. A large set of worker characteristics and plant attributes overlaps between the BIBB-BAuA survey and the LIAB records. We use these common variables to conduct imputations in both possible directions: task information from the BIBB-BAuA survey into LIAB in one direction, and plant-level information from LIAB into BIBB-BAuA in the alternate reverse direction.

For much of our plant-level analysis, the imputation of BIBB-BAuA task information into LIAB is most important. To combine BIBB-BAuA task information with the LIAB plant-worker data and preserve within-occupation and time variation with possibly much precision, we opt for regression-based imputation. Note that the imputation is based on the empirical covariation between common worker variables in both data sets and the tasks that the workers report in BIBB-BAuA, and this covariation preserves the statistically relevant task-related information from BIBB-BAuA in the LIAB data. We first run a linear (OLS) model on the BIBB-BAuA data, regressing the number of tasks (the sum over the 15 activity task indicators) on a set of worker, occupation and plant attributes that are jointly observed in the BIBB-BAuA and in the LIAB data. With the estimated coefficients at hand we perform an out-of-sample linear prediction in the LIAB data using all common variables. Under this procedure, we obtain for 76% of the LIAB observations, an individual-specific activity task count. Finally, by computing the mean over all individuals within a plant, we end up with a measure of the (mean) number of tasks that workers perform within a certain plant (called $b$ in our model below). As shown in the Table 3, the average number of tasks at the plant level varies between 0.32 and 8.87, with a mean of 3.96 and a standard deviation of 0.01.\footnote{The independent variables used in the regression are log daily wage, job experience, squared job experience together with indicators for (i) gender, (ii) 7 schooling and vocational training indicators, (iii) 16 regions, (iv) 34 sectors, (v) 7 plant-size categories, and (vi) 335 occupations. In the baseline regression we pool over the years 1992, 1999, 2006 and 2012. In an alternative specification we estimate the number of tasks separately for these four years and compute year-specific predictions from a moving average.}

\footnote{In the BIBB-BAuA data the average number of tasks for 7 different plant-size categories varies between 4.77 and 5.32, with a mean of 4.92 and standard deviation 0.2. The differences in the task number intervals are mainly due to differences in the wage levels and perhaps the fact the BIBB-BAuA only covers workers with more than 20 hours of work per week.}
Table 3: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>Mean</th>
<th>Median</th>
<th>STD.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>log Revenues</td>
<td>116,931</td>
<td>13.98</td>
<td>13.76</td>
<td>0.01</td>
<td>8.88</td>
<td>24.63</td>
</tr>
<tr>
<td>log Export revenues</td>
<td>36,473</td>
<td>17.48</td>
<td>17.33</td>
<td>0.03</td>
<td>10.92</td>
<td>29.01</td>
</tr>
<tr>
<td>Export indicator</td>
<td>116,933</td>
<td>0.17</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Employment</td>
<td>116,933</td>
<td>18.48</td>
<td>6</td>
<td>0.12</td>
<td>3</td>
<td>44.419</td>
</tr>
<tr>
<td>log Daily wage</td>
<td>116,933</td>
<td>4.13</td>
<td>4.14</td>
<td>0</td>
<td>1.96</td>
<td>5.76</td>
</tr>
<tr>
<td>CV Daily wage</td>
<td>116,933</td>
<td>0.32</td>
<td>0.31</td>
<td>0</td>
<td>0</td>
<td>4.02</td>
</tr>
<tr>
<td>StDev Residual daily wage</td>
<td>116,933</td>
<td>23.07</td>
<td>19.82</td>
<td>0.19</td>
<td>0</td>
<td>1,167.85</td>
</tr>
<tr>
<td>Nr. of 2-digit occupations</td>
<td>116,933</td>
<td>3.5</td>
<td>2</td>
<td>0.01</td>
<td>1</td>
<td>63</td>
</tr>
<tr>
<td>Average number of tasks $b$</td>
<td>116,933</td>
<td>3.96</td>
<td>3.91</td>
<td>0.01</td>
<td>0.32</td>
<td>8.87</td>
</tr>
<tr>
<td>Normalized number of tasks $b/z$</td>
<td>116,933</td>
<td>0.36</td>
<td>0.36</td>
<td>0</td>
<td>0.03</td>
<td>0.70</td>
</tr>
</tbody>
</table>


Note: Descriptive statistics based on annual plant observations, using inverse probability weights to make plant sample representative of Germany economy, as suggested by the Research Data Centre at the IAB. CV is coefficient of variation of observed daily wage. StDev Residual daily wage measures the standard deviation of the (exponentiated) daily (log) wage residual from a Mincer regression (in logs), including demographic, education and tenure information as well as time, sector and region fixed effects and plant revenues.

In addition to mapping information on the number of tasks from BIBB-BAuA to LIAB, we can also estimate the probability of performing a specific task in BIBB-BAuA and make an out-of-sample prediction regarding the probability that an individual worker perform this specific task in the LIAB data. For this purpose, we run 15 probit regressions (one for each task) with the same set of explanatory variables as in the regression for the number of tasks outlined above. With these out-of-sample predictions at hand, we can then construct a measure for the overall number of distinct tasks performed at a plant in LIAB. Due to the chosen estimation approach, the total number of distinct tasks must be smaller than 15 and it is larger than zero if our mapping was successful for at least one worker at the plant.\textsuperscript{13} We then divide the average number of tasks ($b$) by the total number of distinct tasks observed (denoted with $z$ in the model below), to obtain a normalized measure of the number of tasks—a real number on the unit interval: $b/z \in (0,1]$. As shown in the Table 3, the normalized number of tasks $b/z$ varies between 0.03 and 0.7 with a mean of 0.36 and a tight standard deviation.

Table 3 also reports summary statistics on revenues and other relevant plant attributes from the combined LIAB and BIBB-BAuA data. Excluding plants for which we lack relevant information as well as plants with an employment level smaller than or equal to two (for which we cannot compute reasonable measures of wage dispersion), our sample covers 116,931 plant-year observations, with 36,473 of these

\textsuperscript{13}The total number of distinct tasks varies between a minimum of 3.58 and a maximum of 15, with a mean of 11.2 and standard deviation 0.02.
For robustness checks, and for our envisaged worker-level analysis, we propose to also conduct the alternate reverse imputation of plant information from LIAB into the BIBB-BAuA survey data. For this reverse imputation, we predict plant-occupation attributes at the worker level in the LIAB records, using linear regression on the common worker variables, and then employ out-of-sample predictions at the worker level in the BIBB-BAuA survey data to impute plant-occupation attributes—including revenues, export status, occupation counts and within-plant-occupation wage dispersion—into the BIBB-BAuA data at the worker level.

2.5 Main facts

Three robust facts emerge from our data and their combination. First, larger plants and exporters assign their worker to more occupations—an intuitive fact that the LIAB records show clearly in mean comparisons, regressions, and plots.

Second, workers at larger plants and exporters perform fewer tasks within occupations. This second fact emerges from the BIBB-BAuA survey. A worker’s survey response regarding the tasks that she or he performs relates, by design, to the worker’s individual workplace within the plant-occupation. The BIBB-BAuA survey data therefore suffice to establish the second fact. Panel A in Figure 1 shows the

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**Figure 1**: Number of Tasks and Residual Wage Inequality within Plant-Occupation

observations referring to exporters.

Source: BIBB-BAuA 2012 for both panels and LIAB 2012 for panel B.

Notes: Panel A shows the prediction of the number of tasks $b$ within occupation by plant size, controlling for sector, region, occupation and worker characteristics. Panel B shows the prediction of the imputed coefficient of variation of the daily wage residual (exponentiated Mincer residual) $CV$ within plant-occupation by plant size, controlling for sector, region, occupation and worker characteristics. Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.
standardized number of tasks that workers perform at plants of different sizes, after we remove sector, region, occupation and common worker characteristics from the number of tasks through linear regression. The plant size categories in the BIBB-BAuA survey are 1-4, 5-9, 10-49, 50-99, 100-499, 500-999, and 1000 or more workers. The figure plots the number of tasks relative to the baseline size of 1-4 workers against the remaining size categories. There is a clear negative relationship between employer size and the number of tasks within plant-occupations. Larger plants behave more like Adam Smith’s pin factory, assigning narrower task ranges to their workers, up to a size of about 500 workers. Workers at plants with more than 100 workers conduct on average more than 0.3 fewer tasks than workers in the smallest size category with 1 to 4 workers. The median worker in the smallest size category performs 3 tasks, so a task shift by 0.3 tasks amounts to a decrease by more than 10 percent, when workers are employed by plants from the largest three size categories. From the threshold of about 500 workers on, plants assign roughly similar task ranges to their workers. (In the theory, we can capture the constancy beyond the threshold with prohibitively fast increases in fixed costs from the span-of-control problem at large plants.)

Third, overall and residual wages within plant-occupations are more dispersed at larger plants. This third fact emerges from the combined BIBB-BAuA survey data and LIAB records. We compute the coefficient of variation of the (exponentiated) residual log daily wages within plant-occupations across individual workers in the LIAB records, for plants with at least two workers. We then base Panel B in Figure 1 on the reverse imputation of this plant attribute (the coefficient of variation of residual daily wages at the plant) from LIAB into the BIBB-BAuA data. There is a clear positive relationship between employer size and residual wage dispersion within plant-occupations. A similarly strong positive relationship also governs overall wages. In other words, plants that behave more like Adam Smith’s pin factory in the left panel also exhibit more wage dispersion within plant-occupations. Our theory is devised to relate the more pronounced within plant-occupation wage dispersion back to the plant’s internal division of labor (its internal labor market organization).

3 A Model of Production with Task Assignment

3.1 Consumers

We consider an economy with a population of \( L \) individuals, who are risk neutral. As consumers, the individuals have homothetic preferences over a continuum of differentiated goods labelled \( \omega \in \Omega \). The
representative consumer maximizes utility

\[ U = \left[ \int_{\omega \in \Omega} x(\omega)^{\frac{\sigma - 1}{\sigma}} \, d\omega \right]^{\frac{\sigma}{\sigma - 1}} \]  

subject to the economy-wide budget constraint \( \int_{\omega \in \Omega} p(\omega)x(\omega) \, d\omega = Y \), where \( p(\omega) \) is the price of variety \( \omega \), \( Y \) is aggregate income, and \( \sigma > 1 \) is the elasticity of substitution between consumed varieties. The resulting economy-wide demand for variety \( \omega \) of the consumption good is:

\[ x(\omega) = \left( \frac{p(\omega)}{P} \right)^{-\sigma} \frac{Y}{P}, \]  

where \( P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} \, d\omega \right]^{1/(1-\sigma)} \) is the CES price index. A producer of variety \( \omega \) faces total demand \( x(\omega) \) for its product. We introduce heterogeneity in individual consumers’ budget sets given differentiated individual wages below.

### 3.2 Task assignments to occupations and the resulting worker efficiency

Each variety of the consumption good is produced by a unique monopolistically competitive plant. Labor is the only input. As workers, the individuals are endowed with one unit of labor, which they supply inelastically to plants. The individuals differ in their core ability as workers.

Production requires that workers perform tasks in their respective occupations. A plant \( \omega \) decides about three types of employment outcomes. First, the plant chooses the total number of occupations \( n(\omega) + 1 \) that it wants to offer (a plant’s count of occupations in the data). We consider the possible number of occupations \( \lfloor n(\omega) + 1 \rfloor = 1, 2, \ldots \) to be countable and require a plant to offer at least one occupation—when \( n(\omega) = 0 \). Second, the plant assigns an occupation-invariant measure of tasks \( b(\omega) \) that need to be performed within each occupation at the plant. For tractability, we make the total measure of tasks \( b(\omega) \) a real number. By the technology we propose, the first choice of the total number of occupations \( n(\omega) + 1 \) will inversely determine the measure of tasks \( b(\omega) \) as an outcome at the plant level. And third, the plant chooses a measure of workers \( \ell(\omega) \) to hire into the occupations that it offers.

A plant \( (\omega) \) with elemental productivity \( \varphi(\omega) \) produces quantity \( q(\omega) \) of its variety by combining the individual outputs \( q_j(\omega) \) of its occupations \( j = 1, \ldots, n(\omega) + 1 \) into a Cobb-Douglas production function:

\[ q(\omega) = \varphi(\omega) z \lfloor n(\omega) + 1 \rfloor \exp \left[ \frac{1}{n(\omega) + 1} \sum_{j=1}^{n(\omega) + 1} \ln q_j(\omega) \right], \]  

15
where $q_j(\omega)$ is the output of occupation $j$ and $n(\omega) + 1$ is the number of distinct occupations at the plant.\textsuperscript{14} The parameter $z < 1$ is the same across all plants and standardizes the production quantity relative to the number of occupations $n(\omega) + 1$ that the plant offers. The way in which the term $n(\omega) + 1$ enters the production function implies that, in the case of symmetric occupations, plants can raise their output by creating additional occupations, with an elasticity of one. Therefore, worker efficiency does not change in our model just because a plant adds new occupations. Only if workers get to specialize on a smaller range of tasks when new occupations are added, then worker efficiency increases with the addition of these occupations (see below).\textsuperscript{15} To simplify notation for now, we suppress the variety label $\omega$ and consider a single plant.

Workers have innate abilities to carry out tasks more or less efficiently. Complementing the knowledge view of the firm, there are no organizational hierarchies in our model. Instead, the worker abilities are horizontally differentiated and uniformly distributed over a circle with circumference 1. This ability circle simultaneously represents the technology space and characterizes the set of distinct possible tasks, which are also uniformly distributed with measure one. The location of a worker on the circle indicates the task that corresponds to his/her core ability.

Plants cannot use the full circle of tasks for production. Instead they are restricted to select tasks from a subinterval with maximal length $z < 1$. In our data work, we do not observe a plant that is predicted to conduct all 15 tasks (see Section 2.4). The length of the maximally feasible task range $z < 1$ is exogenous and common to all plants. However, plants have a choice to bundle adjacent tasks into occupations, which are then executed by the workers hired for these occupations. Plants must choose a common number (measure) of tasks $b < z$ for all occupations that they offer. In other words, we impose that plants choose symmetric divisions of the segment of the task circle on which they operate. We choose this restriction to reduce the number of parameters to estimate, so our focus in this paper rests on the average number tasks performed per occupation within a plant. We leave potential plant-and-occupation-specific task range choices for future work and start our analysis with only plant-specific task range choices. The measure of tasks $b$ that a plant adopts is thus a constant across all of the plant’s occupations.

Suppose for a moment that tasks on the plant’s segment of the unit circle are mutually exclusively assigned to $n$ separate occupations with no overlap. The plant’s chosen number of occupations $n$ and its

\textsuperscript{14}Output in equation (3) corresponds to a Cobb-Douglas production function of the form $q = \varphi z \prod_j (q_j/\alpha_j)^{\alpha_j}$, with $\sum \alpha_j = 1$ and $\alpha_j = 1/(n + 1)$ under symmetry.

\textsuperscript{15}A generalization to a non-unitary elasticity does not result in substantively different model predictions but would make the model more complicated and would require an additional parameter to discipline with data.
chosen measure of tasks $b$ would then be linked to each other according to

$$b = \frac{z}{n + 1}.$$  

However, in practice and in our data, the same activity (what) tasks are typically performed across multiple occupations. It is arguably the nature of those occupations that the same tasks are repeatedly carried out in occupations in practice, so we introduce an exogenous degree of overlap $\nu$—a common parameter beyond a plant’s control. With a common degree of overlap, a plant’s measure of tasks per occupation becomes

$$b = \frac{z}{\nu n + 1}, \quad \text{with} \quad \nu \in (0, 1]. \quad (4)$$

If $\nu = 1$, eq. (4) collapses to the simple case above with no overlap. If $\nu < 1$, the mapping of tasks to occupations is not unique and the task intervals overlap. In the limiting case of $\nu = 0$, each occupation uses the whole range of tasks of the plant, irrespective of $n$. A symmetric division of the task interval also implies that the interval of tasks covered by an occupation is centered at\(^{16}\)

$$\text{cent}_j = \frac{z}{2} + \frac{(j - 1)2\nu}{\nu n + 1}, \quad (5)$$

with the center being plant-specific because, as discussed below, plants differ in their endogenously chosen count of occupations and because the task intervals characterizing the production processes are drawn from different parts of the technology circle.

Workers have to allocate the same amount of time to all tasks specified by the occupation, with worker efficiency falling in the distance of their core ability to a task. We can therefore interpret the average distance of a worker at location $i$ to the various tasks in interval $[0, b]$ within each occupation as a measure of mismatch. Provided that labor is not misallocated so that workers have their core ability in one of the tasks spanned by the occupation (see below), we can compute a linear measure of mismatch $m(i, b)$ of a worker $i$ with the $b$ tasks in an occupation according to

$$m(i, b) = \frac{1}{b} \left\{ \int_0^i (i - t) \, dt + \int_i^b (t - i) \, dt \right\} = \frac{b^2 + 2i(i - b)}{2b}, \quad (6)$$

where $t$ gives the task location. The mismatch depends on the worker’s position in the interval and is

\(^{16}\)The left-most occupation on interval $[0, z]$ is centered at $b/2$, whereas the right-most occupation is centered at $z - b/2$. All other occupations are symmetrically located between these two ones. We can therefore characterize the center of occupation $j$ by $\text{cent}_j = b/2 + (z - b)(j - 1)/n$, which in view of eq. (4) establishes eq. (5).
lowest (highest) if the worker is located in the middle (at the borders) of the task interval. Intuitively, there is an inverse link between mismatch $m(i, b)$ and a worker $i$’s efficiency $\lambda(i, b)$, which we define as

$$\lambda(i, b) \equiv \frac{\lambda_0}{z} + \frac{1}{m(i, b)} = \frac{\lambda_0}{z} + \frac{2b}{b^2 + 2i(i - b)}.$$  

(7)

For worker efficiency to be well defined, we impose that the magnification parameter $\lambda_0$ satisfies $\lambda_0 > -2z$, so that all workers from interval $[0, b]$ have positive efficiency for all possible outcomes $b \leq z < 1$. The constant magnification parameter $\lambda_0$ will play an important role below when it comes to the intra-plant dispersion of wages and how that wage dispersion varies between plants with different productivities. Note that we do not restrict $\lambda_0$ to be positive.

The plant can choose to hire a measure $\ell_j(i, b)$ of workers with ability $i$ into occupation $j$ given a task range $b$ per occupation. Average worker efficiency in occupation $j$ is then

$$\lambda_j(b) = \frac{1}{\ell_j(b)} \int_0^b \lambda(i, b) \ell_j(i, b) \, di,$$

where

$$\ell_j(b) = \int_0^b \ell_j(i, b) \, di$$

(8)

denotes the total amount of labor hired for occupation $j$. Differentiation of eq. (8) with respect to the task range $b$ per occupation yields

$$\lambda_j'(b) = \frac{\ell_j(b, b)}{\ell_j(b)} [\lambda(b, b) - \lambda_j(b)] < 0.$$  

(9)

This negative relationship between the average worker efficiency in a plant-occupation $j$ and the task range $b$ per plant-occupation is a characterization of the benefits from the division of labor. The relationship provides the theoretical rationale for our second fact (Panel A in Figure 1), by which larger plants adopt narrower task ranges per occupation. As Adam Smith’s pin factory tenet suggests, the efficiency of workers employed in a plant’s occupations is higher when occupations are more specialized in narrower task ranges, because workers are more able on average (closer to their core ability) when conducting fewer tasks in their occupation.

### 3.3 Production

Plants face a going (market clearing) wage rate $w$ for labor ex ante (before worker ability is revealed). Plants differ in the elemental productivity parameter $\varphi$, which they draw from a common Pareto distribution $G(\varphi) = 1 - \varphi^{-g}$ with shape parameter $g$, where $g > 1$ to ensure a finite, positive mean of
productivity. In this lottery, plants also draw the center of their task interval from a uniform distribution over the technology circle. To participate in the lottery, plants hire $f_e$ workers at the going wage rate $w$. After the lottery, the cost is sunk and, depending on its elemental productivity draw, a plant decides on whether to start production. Production requires the additional fixed employment of $f$ workers for overhead services in operation, including the division of tasks into occupations. Plants that adopt many occupations reduce the average width of the task range per occupation so that their workers will be more efficient, compounding the elemental productivity with additional worker efficiency. Once plants have made these entry decisions, they hire the labor input needed for production. Labor is therefore employed in three different roles: for the sunk cost to make the productivity draw $f_e$, for the fixed input into overhead services $f$ to manage and coordinate the occupations, and for the variable input into production. In the first two roles, workers have an efficiency of one, whereas in the third role their efficiency is given by $\lambda(i, b)$ and thus match specific.

To hire workers for production, plants post occupations in a competitive labor market at the going wage $w$. The occupation posting provides an imprecise (binary) signal that informs workers about whether their core ability is within the occupations’s task interval, or not, but not on their specific location within this interval. One way to think about this is that $cent_j$ is not part of the occupation description but they can receive a costless test report that reveals with certainty whether or not their ability is within the occupation’s task range. The occupation posting does specify what the wage schedule will be for the worker upon accepting the occupation offer. Given their risk neutrality, workers will accept any wage schedule that pays an expected wage rate $w$. If production workers can choose an effort level $e$ from interval $[0, 1]$ and thus the time productively used in their occupation (or when providing the fixed inputs), the output of worker $i$ in occupation $j$ is given by

$$q_j(i) = e(i)\lambda(i, b_j).$$

Suppose the utility of workers is reduced by a constant factor $\varepsilon > 0$ per unit of effort. Then plants will link wage payments to the ex post output if the effort is unobservable for outsiders and hence not contractible. The lacking contractibility of effort rules out a uniform wage for all production workers. In fact, plants cannot do better than setting

$$w(i, b) = w\frac{\lambda(i, b)}{\lambda(b)}$$  \hspace{1cm} (10)
for constant going wage $w$, prompting workers to provide full effort $e = 1$ if $\varepsilon$ is sufficiently small. Following this reasoning, we assume that plants pay a constant wage per efficiency unit of $w/\lambda(b)$ to all of their production workers (and $w$ to workers providing fixed inputs), implying that the efficiency differences of workers in the occupation translate one-to-one into wage differences between workers.$^{17}$

Graph A of Figure 2 illustrates the wage dispersion within a plant-occupation that spans a task range $b_0$, where $w_j(i, b) = w\lambda(i, b)/\lambda_j(b)$ under eqs. (7) and (8). Now suppose the plant optimally adopts a narrower task range $b_1 < b_0$ as depicted in Graph B of Figure 2. The wage schedule will still vary around the unchanged economy-wide wage $w$, but it depends on the magnification parameter $\lambda_0$ whether the worker efficiency dispersion, and hence the wage dispersion around the economy-wide mean, stays constant, rises, or falls. For a positive parameter $\lambda_0 > 0$, a narrower task range $b_1 < b_0$ magnifies the worker efficiency dispersion (hence our label magnification parameter for $\lambda_0$) and larger plants with narrower task ranges will exhibit a wider wage dispersion within plant-occupation. We consider it an empirical matter how task ranges should relate to wage outcomes across workers within a plant-occupation and therefore introduce the parameter $\lambda_0$ for estimation. In practice, workers with badly matched abilities near the boundary of a narrow task range might exhibit a more than proportionally diminished efficiency, if their mistakes on the job can result in heavier losses to the employer than in wider task ranges. A priori, it is equally conceivable that badly matched workers in narrow task ranges suffer only a less than propor-

$^{17}$With the wage schedule in eq. (10), there are workers who earn less than the going wage rate $w$. These workers would benefit from quitting and searching for a new occupation elsewhere, as in expectation a new occupation that covers their core ability would offer a higher payment. Whereas this is not a problem in a static setting, if worker efficiency is revealed ex post, one could extend the model to a variant with involuntary unemployment due to search frictions to ensure that quitting remains unattractive for workers even after their efficiency has been revealed. We are preparing an according extension.
tional reduction in efficiency, compared to their efficiency in wide task ranges, if their mistakes matter little to the employer, because narrower task ranges may have a lesser impact on overall production.

Plants pay the same wage per efficiency unit of labor. Plants are therefore indifferent between all applicants. Furthermore, workers are ex ante indifferent between all occupations that correspond to their qualification, i.e. all occupations for which their core ability lies within the covered task interval. Plants therefore end up hiring workers whose abilities are uniformly distributed over the task intervals covered by their occupations. As a result, average worker efficiency is the same for all occupations in the plant and given by

\[
\lambda(b) = \frac{1}{b} \int_0^b \lambda(i, b) \, di = \frac{1}{b} \left[ \frac{i \lambda_0}{z} + 2 \arctan \left( \frac{2i - b}{b} \right) \right]_0^b = \frac{\lambda_0}{z} + \frac{\pi}{b} = \frac{1}{z} [\lambda_0 + \pi(\nu n + 1)],
\]

where the last equality sign follows when substituting \( z/b = \nu n + 1 \) from eq. (4). It follows from these insights that the wage dispersion is the same in all occupations of a plant and linked to the plant’s chosen task range per occupation with

\[
\text{var}(b) = \frac{w^2}{b} \int_0^b \left( \frac{\lambda(i, b)}{\lambda(b)} \right)^2 \, di - w^2 = w^2 \frac{[4 + \pi(2 - \pi)](\nu n + 1)^2}{[\lambda_0 + \pi(\nu n + 1)]^2}.
\]

### 3.4 Profit maximization in the closed economy

Plants decide about entry and production in three stages. On stage one, a plant \( \omega \) decides on paying the sunk cost of \( f_c \) units of labor for entering the elemental productivity draw. On stage two, the plant decides on starting production conditional on its productivity draw. Prior to production on stage three, the plant must also determine on stage two the number of occupations \( n(\omega) \) and pay a fixed cost of \( f(\omega) \) units of labor to operate. We set the plant’s fixed cost of operation to

\[
f(\omega) = f_0 + \left\{ \lambda_0 + \pi([\nu n(\omega) + 1]) \right\}^\gamma
\]

with a semi-elasticity of the fixed cost with respect to occupation counts \( \gamma > 0 \), so that the overhead costs are positively linked to the count of occupations \( n(\omega) \) at the plant. It is costly to the plants to create additional occupations (and have a narrower task range per occupations). On stage three, plants hire production workers \( \ell(\omega) \), manufacture output \( q(\omega) \) and sell this output to consumers. We solve the three-stage decision problem by backward induction.
On stage three, a plant sets $\ell_j(\omega)$ to maximize its profits

$$
\psi(\omega) = p(\omega)q(\omega) - w \sum_{j=1}^{n(\omega)+1} \ell_j(b(\omega)) - w \{\lambda_0 + \pi[n(\omega) + 1]\}^\gamma - w f_0,
$$

subject to aggregate consumer demand for their variety (2), the market clearing condition $x(\omega) = q(\omega)$ for their variety, and the plant’s production function

$$
q(\omega) = \varphi(\omega)[n(\omega) + 1] \{\lambda_0 + \pi[n(\omega) + 1]\} \exp \left[ \frac{1}{n(\omega) + 1} \sum_{j=1}^{n(\omega)+1} \ln \ell_j(b(\omega)) \right],
$$

under a set of common non-negativity constraints. Profit maximization on stage three results in the first-order condition for revenues and employment

$$
r(\omega)[(\sigma - 1)/\sigma] = [n(\omega) + 1]w\ell_j(b(\omega)),
$$

with $r(\omega) \equiv p(\omega)q(\omega)$. This first-order condition establishes the intuitive result that plant $\omega$ chooses the same employment level for all occupations $j$: $\ell_j(b(\omega)) = \hat{\ell}(\omega)$. Furthermore, the profit-maximizing price can be expressed as a constant markup over the plant’s marginal cost $p(\omega) = [\sigma/(\sigma - 1)]c(\omega)$, given CES demand, with

$$
c(\omega) \equiv \frac{w}{\varphi(\omega) \{\lambda_0 + \pi[n(\omega) + 1]\}}.
$$

(15)

We now turn to stage two. Plants rationally anticipate the profit value on stage three as a function of their entry decisions and choice of the count of occupations. Substituting eq. (15) into eq. (13) and accounting for eq. (2) yields profits of plant $\omega$ as a function of the count of occupations chosen by the plant $n(\omega)$:

$$
\psi(\omega) = \frac{Y}{P^{1-\sigma}} \frac{1}{\sigma - 1} \left[ \frac{\sigma}{\varphi(\omega) \{\lambda_0 + \pi[n(\omega) + 1]\}} \right]^{1-\sigma} - w \{\lambda_0 + \pi[n(\omega) + 1]\}^\gamma - w f_0.
$$

(16)

Plants face the trade-off that increasing the count of occupations lowers marginal costs with a positive effect on profits, but at the same time raises the overhead costs with a negative effect on profits. This trade-off is similar to the one in Eckel (2009) and Bustos (2011), where producers can pay a fixed cost to reduce variable production costs.

Treating $n(\omega)$ as a continuous variable for purposes of exposition, the first-order condition for the
profit-maximization problem at stage two is given by:

\[ r(\omega) \frac{\sigma - 1}{\sigma} = \gamma w \{ \lambda_0 + \pi [\nu n(\omega) + 1] \}^\gamma. \]  

(17)

We assume that \( \gamma > \sigma - 1 \), a necessary condition for an interior solution to be a maximum. In addition, we assume that parameters are such that every plant benefits from specifying more than one occupation, i.e. from setting \( n(\omega) > 0 \). Think of plants with at least one worker in a more organizational (senior) role and another worker in a more operational (junior) role. The plant that gains least from increasing \( n(\omega) \) is the plant with the lowest \( \varphi(\omega) \). This is the plant that makes zero profits from production \( \hat{\psi}(\omega) = 0 \), provided that not all plants find it attractive to start production (see below). In an interior maximum, this zero-profit condition can be expressed as

\[ \frac{f_0(\sigma - 1)}{\gamma - \sigma + 1} = \{ \lambda_0 + \pi [\nu n(\omega) + 1] \}^\gamma, \]  

(18)

and hence we can safely conclude that the maximization problem has an interior solution if eq. (18) holds for a strictly positive \( n(\omega) \), that is for \( f_0(\sigma - 1)/[\gamma - \sigma + 1] > (\lambda_0 + \pi)^\gamma \). This latter inequality characterizes the parameter domain to which we restrict ourselves because, in combination with \( \gamma > \sigma - 1 \), it is sufficient for a unique maximum at stage two, with \( n(\omega) > 0 \) for all plants.

Eqs. (2) and (15) together with market clearing condition \( x(\omega) = q(\omega) \) establish a first relationship between relative revenues of two plants and the relative number of distinct occupations in these plants, while eq. (17) establishes a second relationship between these variables. We have:

\[ \frac{r(\omega_1)}{r(\omega_2)} = \left( \frac{\varphi(\omega_1) [\lambda_0 + \pi [\nu n(\omega_1) + 1]]}{\varphi(\omega_2) [\lambda_0 + \pi [\nu n(\omega_2) + 1]]} \right)^{\sigma - 1}, \quad \frac{r(\omega_1)}{r(\omega_2)} = \left( \frac{\lambda_0 + \pi [\nu n(\omega_1) + 1]}{\lambda_0 + \pi [\nu n(\omega_2) + 1]} \right)^{\gamma}. \]  

(19)

respectively. These expressions allow us to express relative revenues and the relative count of occupations of two plants as a function of these plants’ relative differences in their elemental productivity parameter \( \varphi \):

\[ \frac{r(\omega_1)}{r(\omega_2)} = \left( \frac{\varphi(\omega_1)}{\varphi(\omega_2)} \right)^{\xi}, \quad \frac{\lambda_0 + \pi [\nu n(\omega_1) + 1]}{\lambda_0 + \pi [\nu n(\omega_2) + 1]} = \left( \frac{\varphi(\omega_1)}{\varphi(\omega_2)} \right)^{\xi}, \]  

(20)

where \( \xi \equiv \gamma(\sigma - 1)/(\gamma - \sigma + 1) \) denotes the elasticity of revenues with respect to productivity parameter \( \varphi \). Using eqs. (2) and (14), we can also determine relative output and relative plant-level employment of
production workers as a function of the productivity differential between these plants:

\[ \frac{q(\omega_1)}{q(\omega_2)} = \left( \frac{\varphi(\omega_1)}{\varphi(\omega_2)} \right)^{\frac{\xi_0}{\gamma}} , \quad \frac{\ell(\omega_1)}{\ell(\omega_2)} = \left( \frac{\varphi(\omega_1)}{\varphi(\omega_2)} \right)^{\xi} , \]  

(21)

where \( \ell(\omega) \equiv [n(\omega) + 1] \hat{\ell}(\omega) \) is the employment of production workers at plant \( \omega \). Our framework shares with other models of heterogeneous employers the empirically well-documented property that workers in larger firms are more productive (see Idson and Oi 1999). However, in contrast to other contributions, the productivity differences are further scaled up under the plants’ profit maximizing choice of the count of occupations, which raises worker efficiency.

It is an important insight from eqs. (20) and (21) that plant outcomes are fully characterized by exogenous differences in \( \varphi \). Hence, we can drop \( \omega \) and index plants by their elemental productivity parameter from now on.

Denoting productivity of the marginal plant by \( \varphi^* \), revenues of and the count of occupations in the marginal plant are given by

\[ r(\varphi^*) = \frac{\sigma \xi w f_0}{\sigma - 1} , \quad \nu n(\varphi^*) + 1 = \frac{1}{\pi} \left[ \left( \frac{\xi f_0}{\gamma} \right)^{1/\gamma} - \lambda_0 \right] , \]  

(22)

respectively, according to eqs. (17) and (18). Furthermore, the variance of wages within occupations for the marginal producer can be expressed as

\[ var(\varphi^*) = \left[ 4 + \pi (2 - \pi) \right] \left( \frac{w}{\pi} \right)^2 \left[ \frac{(\xi f_0/\gamma)^{1/\gamma} - \lambda_0}{(\xi f_0/\gamma)^{1/\gamma}} \right]^2 , \]  

(23)

according to eqs. (12), (18), and (22). The variance of wages in plants with \( \varphi > \varphi^* \) is then given by

\[ var(\varphi) = var(\varphi^*) \left[ \frac{(\varphi / \varphi^*)^{\xi/\gamma} (\xi f_0/\gamma)^{1/\gamma} - \lambda_0}{(\varphi / \varphi^*)^{\xi/\gamma} (\xi f_0/\gamma)^{1/\gamma} - \lambda_0} \right]^2 , \]  

(24)

according to eqs. (17), (20), and (23). It is easily confirmed from eq. (24) that the variance of wages is the same for all producers and thus independent of productivity parameter \( \varphi \) if \( \lambda_0 = 0 \). In contrast, the variance of wages strictly increases in \( \varphi \) if \( \lambda_0 > 0 \), and weakly decreases otherwise.

To solve for the plants’ problem at stage one, we note that free entry is consistent with profit maximization if and only if the expected profit from participating in the productivity draw, \( \int_{\varphi^*}^{\infty} \psi(\varphi) dG(\varphi) \), just compensates plants for the sunk costs of participation, \( w f_e \). As shown in the Appendix, we have
\[
\int_{\varphi}^{\infty} \psi(\varphi) \, dG(\varphi) = [1 - G(\varphi^*)] w f_0 \xi / (g - \xi),
\]
which allows us to solve for the productivity of the marginal plant:
\[
\varphi^* = \left( \frac{f_0}{f_e} \frac{\xi}{g - \xi} \right)^{\frac{1}{\gamma}}
\tag{25}
\]
that participates in the productivity draw, where we assume \( g > \xi \) to ensure a positive and finite value of aggregate revenues and profits, and we assume \( f_0 / f_e > g / \xi - 1 \) to ensure \( \varphi^* > 1 \) and hence an outcome by which only relatively more productive plants start production at stage two.

### 3.5 The autarky equilibrium

With the insights from the previous subsection, we can now solve for the general equilibrium. For this purpose, we choose labor as the numéraire and set \( w = 1 \). Since profit income is used to pay for entrance into the productivity lottery, the mass of producers is determined by the condition that economy-wide labor income, \( L \), equals total consumption expenditures, \( Y \), and thus aggregate revenues \( R = M r(\varphi^*) g / (g - \xi) \). Accounting for eq. (22), we obtain
\[
M = \frac{L(\sigma - 1) g - \xi}{\sigma \xi f_0 g}.
\tag{26}
\]
Welfare of the representative agent is (proportional to) the real wage and thus given by the inverse of the CES price index: \( W = P^{-1} \). The price index can be expressed as \( P = [g M / (g - \xi)]^{1/(1-\sigma)} p(\varphi^*) \) and it therefore follows from eqs. (15), (25), (26) and constant markup pricing that welfare is given by
\[
W = \left( \frac{L}{\gamma} \right)^{\frac{1}{\gamma}} \left( \frac{\gamma}{f_0 \xi} \right)^{\frac{1}{\gamma}} \left( \sigma - 1 \right)^{\frac{\sigma}{\gamma}} \left( \frac{f_0}{f_e} \frac{\xi}{g - \xi} \right)^{\frac{1}{\gamma}}.
\tag{27}
\]

To complete the discussion of the closed economy, we compute economy-wide wage inequality as the employment-share weighted average of the variances of wages at the plant level:
\[
Var = \frac{g - \xi}{gL(\varphi^*)} \int_{\varphi^*}^{\infty} \text{var}(w, \varphi) \ell(\varphi) \, dG(\varphi),
\tag{28}
\]
where \( \ell(\varphi^*) g / (g - \xi) \) is the average employment of production workers per plant. Solving for the integral gives
\[
Var = \text{var}(\varphi^*) \left\{ 1 + \frac{2\xi / \gamma}{g - \xi + 2\xi / \gamma (f_0 \xi / \gamma)^{1/\gamma}} \right\} \left[ 1 + \frac{(f_0 \xi / \gamma)^{1/\gamma}}{(f_0 \xi / \gamma)^{1/\gamma} - \lambda_0 \frac{\xi / \gamma}{f_0 \xi / \gamma - \lambda_0 g - \xi + 2\xi / \gamma}} \right],
\tag{29}
\]
according to eq. (24). This implies that \( \text{Var} > \text{var}(\varphi^*) \) if and only if \( \lambda_0 > 0 \).

4 Task Assignment in the Open Economy

4.1 Fundamentals

In this section, we consider trade between two symmetric countries with consumption and production as in the previous section. There are two types of trade costs: fixed costs \( f_x > 0 \) (in units of labor) for setting up a foreign distribution network; and variable iceberg transport costs \( \tau > 1 \) with the usual interpretation that \( \tau \) units of the consumption good must be shipped in order for one unit to arrive in the foreign economy. Both of these costs are also present in the Melitz (2003) framework and – in combination with the heterogeneity of plants in their elemental productivity parameter \( \varphi \) – they generate self-selection of only the most productive producers into exporting, provided that the trade costs are sufficiently high. However, compared to other studies the decision to start exporting is more sophisticated in our model, because it influences a plant’s optimal choice of \( n(\varphi) \) and thus exerts a feedback effect on profits attainable in the domestic market. Due to this feedback effect, we have to distinguish between variables referring to exporters (denoted by super-/subscript \( e \)) and non-exporters (denoted by super-/subscript \( d \)). Furthermore, we use subscript \( t \) to refer to variables associated with total (domestic and foreign) market activities.

Holding economy-wide variables constant, access to exporting does not affect the maximization problem of non-exporters. Things are different, however, for exporters, who make revenues \( \tau^{1-\sigma} r^e(\varphi) \) in the foreign market in addition to their revenues \( r^e(\varphi) \) in the domestic market, implying that in the open economy this plants’ profit-maximizing choice of \( n(\varphi) \) is given by

\[
(1 + \tau^{1-\sigma}) r^e(\varphi) \frac{\sigma - 1}{\sigma} = \gamma (\lambda_0 + \pi (1 + \nu n(\varphi)))
\]

instead of eq. (17). Since eq. (30) is structurally the same for all exporters, we can conclude that the ratios depicted by eqs. (20) and (21) remain unaffected in the open economy, provided that the two plants have the same export status (\( d \) or \( e \)). In contrast, when comparing two plants with the same productivity parameter \( \varphi \) but differing export status, we obtain

\[
\frac{r^e(\varphi)}{r^d(\varphi)} = (1 + \tau^{1-\sigma})^{\frac{\xi}{\gamma}}, \quad \frac{\lambda_0 + \pi (1 + \nu n^e(\omega))}{\lambda_0 + \pi (1 + \nu n^d(\omega))} = (1 + \tau^{1-\sigma})^{\frac{\xi}{\gamma}},
\]

(31)
\[
\frac{q_e^e(\varphi)}{q^d(\varphi)} = (1 + \tau^{1-\sigma}) \frac{\xi}{\sigma - 1}, \quad \frac{\ell^e_t(\varphi)}{\ell^d_t(\varphi)} = (1 + \tau^{1-\sigma}) \frac{\xi}{\sigma - 1},
\]

(32)

where \(l^e_t(\varphi)\) and \(l^d_t(\varphi)\) denote total labor input of plant \(\varphi\) under exporting and non-exporting, respectively.

From the analysis of the closed economy, we know that larger plants have more occupations. Since, all other things equal, exporting generates additional revenues from sales to foreign consumers, it leads to a larger count of occupations: \(n^e(\varphi) > n^d(\varphi)\). The stronger division of labor makes exporters more productive and lowers their unit production costs. This stimulates sales in both the domestic and the foreign market, establishing \(r^e(\varphi) > r^d(\varphi)\) and \(q^e(\varphi) > q^d(\varphi)\) in eqs. (31) and (32), respectively. Hence, there is a positive feedback effect of exporting on domestic revenues and this raises the incentives of plants to export ceteris paribus. From eq. (32) we can also infer that exporting changes the hiring decision. Whereas exporters install more occupations and therefore increases productivity of their workforce, the associated rise in efficiency units of labor does not fully compensate for the higher demand of labor from higher sales at home and abroad. Hence, a plant that starts to export also hires additional workers.

Despite the additional complexity arising from the feedback effect that a plant’s exporting decision exerts on its domestic profits, our model preserves key properties of the Melitz (2003) model, regarding the partitioning of plants by export status. To see this, we can make use of (17), (20), (22), (30), and (31) and write a plant’s profit gain from exporting, \(\Delta \psi_t(\varphi) \equiv \psi^e_t(\varphi) - \psi^d_t(\varphi)\), as follows:

\[
\Delta \psi_t(\varphi) = \left(1 + \tau^{1-\sigma}\right) \frac{\xi}{\sigma - 1} - 1 \left(\frac{\varphi}{\varphi^*}\right)^\xi f_0 - f_x.
\]

(33)

The profit differential in (33) increases in the elemental productivity parameter \(\varphi\), and hence there is selection of the most productive plants into exporting as in other applications of the Melitz model if the two trade cost parameters \(f_x\) and \(\tau\) are sufficiently high. This is the case we are focussing on in this paper, and we can therefore characterize the plant that is indifferent between exporting and non-exporting by \(\Delta \psi_t(\varphi) = 0\). We denote the (cutoff) productivity of this plant by \(\varphi^*_x\), implying that plants with \(\varphi \geq \varphi^*_x\) are exporters, while plants with \(\varphi < \varphi^*_x\) are non-exporters. Solving \(\Delta \psi_t(\varphi^*_x) = 0\) for the ratio of the two productivity cutoffs \(\varphi^*_x\) and \(\varphi^*\) and noting that the share of exporters is given by \(\chi \equiv [1 - G(\varphi^*_x)]/[1 - G(\varphi^*)]\), we can compute

\[
\chi = \left\{\frac{f_0}{f_x} \left[ (1 + \tau^{1-\sigma}) \frac{\xi}{\sigma - 1} - 1 \right]\right\}^{\frac{\sigma}{\xi}},
\]

(34)

with \(\chi < 1\) if there is partitioning of plants by their export status. From eq. (34), we can conclude that
higher trade costs, i.e. a higher \( f_x \) or a higher \( \tau \), raise the minimum productivity level that is necessary to render exporting an attractive choice, thereby lowering the share of exporters in the total population of active plants \( \chi \). With this insights at hand, we are now equipped to solve for the open economy equilibrium.

4.2 The open economy equilibrium

Profit maximization in the open economy is described by a three-stage decision problem that is similar to the closed economy, but additionally involves the decision to start exporting or to sell exclusively to the domestic market (at stage 2). Access to the export market raises profits of the most productive plants, and thus the expected profit prior to entry into the productivity lottery, which as shown in the Appendix is given by

\[
\int_{\varphi^*}^{\infty} \psi_t(\varphi) \, dG(\varphi) = \left[ 1 - G(\varphi^*) \right] \frac{\xi f_0}{g - \xi} \left( 1 + \frac{\chi f_x}{f_0} \right)
\]

in the open economy. Free entry into the productivity lottery establishes \( \int_{\varphi^*}^{\infty} \psi_t(\varphi) \, dG(\varphi) = f_e \) and thus \( \varphi^* / \varphi_a^* = (1 + \chi f_x / f_0)^{1/g} \), where index \( a \) is used to indicate an autarky variable. Access to exporting increases expected profits from production, and hence the probability to start production \( 1 - G(\varphi^*) \) must decrease in order to restore the condition of zero expected profits from entry, which is achieved by a higher cutoff productivity level. This mechanism is well understood from Melitz (2003) and points to asymmetric effects of openness at the plant level. Whereas the most productive plants see their profits soaring due to access to the foreign market, low-productivity plants experience a profit loss due to stronger competition (for scarce labor), with the least productive ones being forced to cease production.

To shed further light on the asymmetry in plant-level responses to trade, we can study how producers adjust their assignment of workers to tasks in the open economy. We start with a close look at non-exporting plants. The fixed overhead costs of the marginal producer \( f(\varphi^*) \) remain to be determined by eq. (18). However, the marginal producer in the open economy must have a higher elemental productivity parameter than the marginal producer in the closed economy, and that plant’s fixed costs are therefore lower than under autarky. In view of eq. (20) fixed costs are lower for all non-exporting plants, implying that these plants reduce their count of occupations in response to trade. This is intuitive because non-exporting plants lose market shares in the open economy, and hence find it more difficult to bear the overhead costs associated with the creation of distinct occupations. Contrasting the overhead costs for the division of tasks into occupations in the closed and the open economy, we can compute in the case of
non-exporting plants:

\[
\frac{\lambda_0 + \pi (1 + \nu n^d(\varphi))}{\lambda_0 + \pi [\nu n^a(\varphi) + 1]} = \left(\frac{1}{1 + \chi f_x/f_0}\right) \frac{\xi}{g} \equiv \rho_d(\varphi) < 1,
\]

according to eq. (20) and our previous insight that \(\varphi^*/\varphi^a = (1 + \chi f_x/f_0)^{1/g}\). For exporting plants, we can compute

\[
\frac{\lambda_0 + \pi (1 + \nu n^e(\varphi))}{\lambda_0 + \pi [\nu n^a(\varphi) + 1]} = \left(\frac{1 + \tau^{1-\sigma}}{1 + \chi f_x/f_0}\right) \frac{\xi}{g} \equiv \rho_e(\varphi),
\]

where the first equality sign follows from eqs. (31) and (36), whereas the second one follows from eq. (34). Noting that \(g > \xi\) holds by assumption, it is straightforward to show that \(n^e(\varphi) > n^a(\varphi)\) and thus \(\rho_e(\varphi) > 1\). A plant that starts exporting in the open economy realizes higher revenues and thus raises the number of distinct occupations. It is notable that the asymmetric response of plants in their internal division of labor is the consequence of an asymmetric exposure of this plants to exporting. If all plants would start exporting (\(\chi = 1\)), the marginal plant would be the same as in the closed economy, \(\varphi^* = \varphi^a\), implying \(n^e(\varphi) = n^a(\varphi)\) for all active producers. It is therefore the asymmetric exposure to exporting rather than the market size increase per se that is responsible for plant-level adjustments in the division of tasks into occupations. The following proposition summarizes the insights from the previous analysis.

**Proposition 1** Opening up to trade with selection of the most productive plants into exporting leads to an asymmetric response in the within-plant assignment of workers to tasks. Whereas exporter lower the number of tasks covered by occupations, resulting in lower mismatch and higher worker efficiency, non-exporters increase the task coverage of occupations, resulting in larger mismatch and lower worker efficiency.

The asymmetric response of plants to trade regarding the division of tasks into occupations has consequences for wage differences in those occupations. Following the derivation steps of the closed economy, we can express wage inequality of non-exporters, \(\text{var}_d(w, \varphi)\), as a function of wage inequality in the marginal plant, \(\text{var}_d(w, \varphi^*)\), by eq. (24). Thereby, wage inequality in the marginal plant is the same as in the closed economy. However, in the open economy this is a plant with higher productivity. Accounting for \(\varphi^*/\varphi^a = (1 + \chi f_x/f_0)^{1/g}\) the effect of openness on wage inequality at the plant-level for
non-exporters is therefore determined by

\[\text{var}_d(w, \varphi) = \text{var}_a(w, \varphi) \left[ \frac{(\varphi/\varphi_a^*)^{\xi/g}(\xi f_0/\gamma)^{1/\gamma} - \lambda_0 \rho_a^{-1}(\varphi)}{(\varphi/\varphi_a^*)^{\xi/g}(\xi f_0/\gamma)^{1/\gamma} - \lambda_0} \right]^2, \tag{38}\]

according to eq. (36). Accounting for \( r_d(\varphi) < 1 \), it follows from eq. (38) that \( \text{var}_d(w, \varphi) > \text{var}_a(w, \varphi) \) if and only if \( \lambda_0 < 0 \). Looking at exporting plants, we can compute

\[\text{var}_e(w, \varphi) = \text{var}_a(w, \varphi) \left[ \frac{(\varphi/\varphi_a^*)^{\xi/g}(\xi f_0/\gamma)^{1/\gamma} - \lambda_0 \rho_e^{-1}(\varphi)}{(\varphi/\varphi_a^*)^{\xi/g}(\xi f_0/\gamma)^{1/\gamma} - \lambda_0} \right]^2, \tag{39}\]

according to eqs. (24) and (36). Accounting for \( r_e(\varphi) > 1 \) it therefore follows that \( \text{var}_e(w, \varphi) > \text{var}_a(w, \varphi) \) if and only if \( \lambda_0 > 0 \). The following proposition summarizes the effects of trade on plant-level wage inequality.

**Proposition 2** Opening up to trade with selection of the most productive plants into exporting leads to an asymmetric response in plant-level wage inequality. The variance of wages increases (decreases) in exporting plants if wage inequality was already exceptionally high (low) for these producers under autarky. The opposite is true for non-exporters.

Given the asymmetry in the plant-level implications, access to trade exerts counteracting effects on the general equilibrium variables of interest: welfare \( W \) and economy-wide wage inequality \( \text{Var} \). Similar to autarky, welfare in the open economy is given by the real wage and hence inversely related to the CES price index \( P = [g M(1 + \chi f_x/f_0)/(g - \xi)]^{1/(1-\sigma)} p^a(\varphi^*). \) The mass of producers in the open economy is given by \( M = M_a/(1 + \chi f_x/f_0) \) and thus smaller than in the closed economy. Noting further that \( p(\varphi^*) = p^a(\varphi^*)(1 + \chi f_x/f_0)^{-1/g}, \) we can relate welfare in the open economy to welfare in the closed economy, according to

\[ W = W_a \left( 1 + \frac{\chi f_x}{f_0} \right)^{-\frac{1}{g}}. \tag{40} \]

For plant entry in our model is allocationally efficient (similar to the case in Dhiingra and Morrow 2016), a movement from autarky to trade is akin to lifting a technology barrier, which must be welfare enhancing.

As shown in the Appendix, economy-wide wage inequality in the open economy is given by

\[ \text{Var} = \text{Var}_a + \frac{\lambda_0 \text{var}_d(w, \varphi^*)}{[(\xi f_0/\gamma)^{1/\gamma} - \lambda_0]^2 g - \xi} \frac{\chi^{-\frac{1}{g}}}{g - \xi + 2\xi/\gamma} \frac{1 + \chi f_x/f_0}{V(\chi)}, \tag{41} \]
Economy-wide wage inequality is therefore higher (lower) in the open than the closed economy if and only if \( \lambda_0 > 0 \). If \( \lambda_0 > 0 \), then wage inequality within high-productivity plants increases while wage inequality within low-productivity plants falls. As a consequence, there are counteracting effects on economy-wide wage inequality. However, the combined effect is unambiguous for two reasons. On the one hand, aggregate overhead expenditures associated with the division of tasks into occupations increase, which raises wage inequality if \( \lambda_0 > 0 \). On the other hand, exporters expand production and hire new workers, whereas non-exporters contract production and release part of their workforce. Hence, plants with a larger wage inequality get a higher weight in the computation of \( Var \), which contributes to an increase in economy-wide wage inequality. We summarize the effects of trade on welfare and economy-wide wage inequality in the following proposition.

**Proposition 3** Opening up to trade with selection of the most productive plants into exporting increases welfare. Trade increases (decreases) economy-wide wage inequality if it widens (compresses) wage differences within exporters.

We complete the analysis in this section by shedding light on the consequences of a marginal reduction in transport cost parameter \( \tau \). Such a decline increases the expected income from exporting, and thus raises \( \chi \), according to (34). From eqs. (36) and (37), we can infer \( d\rho_q(\varphi)/d\chi < 0 \) and \( d\rho_e(\varphi)/d\chi > 0 \), and can hence conclude that the effect of declining transport costs on the division of tasks into occupations and its consequences for plant-level worker efficiency and within-plant wage inequality are monotonic. Welfare increases monotonically in \( \chi \), according to eq. (40). However, the impact of a higher \( \chi \) on economy-wide wage inequality needs not be monotonic. The reason is that plants that start to export adjust their number of distinct occupations discretely, and hence the effect on plant-level employment is stronger for new than for incumbent exporters. At high levels of \( \chi \), new exporters are low-productivity plants and, if \( \lambda_0 > 0 \), these are plants with relatively low wage inequality. If employment in these plants increases disproportionately, we conjecture that this change may dominate the overall increase in the number of
distinct occupations across plants, thereby lowering economy-wide wage inequality at high levels of $\chi$. We will explore this possibility with numerical simulations in future drafts and revisit the question after structurally estimating parameters for the German economy.

5 Empirical Tests of the Model

The theoretical model outlined in Sections 3 and 4 gives rise to two testable hypotheses at the plant level:

**Hypothesis 1.** The number of tasks and revenues are inversely related.

**Hypothesis 2.** The within-plant wage dispersion is positively related to plant revenues iff the magnification parameter is positive, $\lambda_0 \geq 0$.

To test these hypotheses and to see whether their patterns are robust, we run a series of regressions, in which we vary the set of explanatory variables and instrument those regressors, whose endogeneity is suggested by our theoretical model. We consider the variables in logs to reduce sensitivity with respect to outliers and to make the results of our analysis more easily accessible to economic interpretation. To guard our estimates against an omitted variable bias, we control in all specification for time, region and sector fixed effects in addition to the explanatory variables listed in the tables. Standard errors are clustered at the plant level, because we observe a large fraction of plants repeatedly.

**Test of hypothesis 1:** Table 4 reports the results from estimating the link between the normalized number of tasks $b/z$ and plant-level revenues. The first three columns of the table present the outcome of OLS regressions, which support our theoretical hypothesis of a negative link between plant-level revenues and the number of tasks. The baseline specification in Column 1 suggests that a ten percent increase in plant-level revenues is associated with a one percent decline in the (normalized) number of tasks $b/z$. This effect gets smaller when we add the log number of distinct occupations in a plant and the interaction term of the log number of occupations and log revenues as further explanatory variables. A negative impact of the number of distinct occupations on the number of tasks is in line with our theoretical model. However, from our model one may expect that the number of occupations and revenues are perfectly correlated, which is not the case.\footnote{The correlation coefficient of these two variables amounts to 0.671.} But this should not be interpreted as evidence against the formal structure of our model because the model does not predict a linear relationship between log revenues (or the log number
of occupations) and the log normalized number of tasks \( b/z \), and hence the fact that we are able to estimate significant effects of all three explanatory variables in Column 2 could simply reflect non-linearities in the relationship between these variables and the log normalized number of tasks. Overall, the marginal effect of an increase in log revenues on the log normalized number of tasks is still negative and amounts to -0.042, when evaluated at the mean of the log number of occupations, 1.648. In Column 3 we see that the negative relationship between revenues and the number of occupations is also robust to adding fixed plant effects.

### Table 4: The impact of revenues on the number of tasks

<table>
<thead>
<tr>
<th>Dependent variable: log Normalized number of tasks ( b/z )</th>
<th>(1) OLS</th>
<th>(2) OLS</th>
<th>(3) OLS</th>
<th>(4) IV</th>
<th>(5) IV</th>
<th>(6) IV</th>
<th>(7) IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>log revenues</td>
<td>-0.091***</td>
<td>-0.057***</td>
<td>-0.051***</td>
<td>-0.021</td>
<td>-0.021*</td>
<td>-0.259***</td>
<td>-0.257***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.009)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.077)</td>
<td>(0.076)</td>
</tr>
<tr>
<td>log nr. of occupations</td>
<td>-0.257***</td>
<td>-0.328***</td>
<td>4.363**</td>
<td>4.428**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.075)</td>
<td>(1.975)</td>
<td>(2.010)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log revenues</td>
<td>0.009***</td>
<td>0.013**</td>
<td>-0.226**</td>
<td>-0.230**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \times ) log nr. of occupations</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.110)</td>
<td>(0.112)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plant FE</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.234</td>
<td>0.244</td>
<td>0.845</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>0.234</td>
<td>0.243</td>
<td>0.793</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hansen J (p-val.)</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.288</td>
<td>n.a.</td>
<td>0.872</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>126,488</td>
<td>126,488</td>
<td>126,488</td>
<td>64,907</td>
<td>64,616</td>
<td>64,777</td>
<td>64,563</td>
</tr>
</tbody>
</table>

Note: The panel covers all industries and regions of Germany between 1996-2014. All regressions include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors in parentheses. Significance levels are indicated by * \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.01 \).

Our theoretical model implies that plant-level revenues depend on the endogenous number of tasks conducted by workers, and since the number of distinct occupations chosen by a plant is positively related to its revenues in our model, we can expect that the explanatory variables in Columns 1-3 are correlated with the error term, leading to biased estimates due to simultaneous causality. To overcome this problem, we use an instrumental variable (IV) approach and estimate the relationship between revenues and the number of tasks (in logs), using GMM. The second-stage results of the respective regressions are reported in Columns 4-7 of Table 4, whereas the first-stage results are summarized in Table 5.

Our choice of instruments is guided by insights from our theoretical model, which predicts that deeper economic integration in the form of more industry exports and more industry imports affects plant-level revenues, the number of distinct occupations, and thus the number of tasks conducted by workers. This
### Table 5: The impact of revenues on the number of tasks – first stage

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(4.1)</td>
<td>(5.1)</td>
<td>(6.1)</td>
<td>(6.2)</td>
<td>(6.3)</td>
<td>(7.1)</td>
<td>(7.2)</td>
<td>(7.3)</td>
</tr>
<tr>
<td>export dum&lt;sub&gt;−1&lt;/sub&gt;</td>
<td>0.116***</td>
<td>0.116***</td>
<td>0.219***</td>
<td>2.428***</td>
<td>0.129***</td>
<td>0.212***</td>
<td>2.373***</td>
<td>0.126***</td>
</tr>
<tr>
<td>× log exports&lt;sub&gt;CHN&lt;/sub&gt;</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.020)</td>
<td>(0.243)</td>
<td>(0.015)</td>
<td>(0.018)</td>
<td>(0.239)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>log imports&lt;sub&gt;CHN&lt;/sub&gt;</td>
<td>-0.017</td>
<td></td>
<td></td>
<td></td>
<td>-0.193***</td>
<td></td>
<td>-1.201***</td>
<td>-0.069***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td></td>
<td></td>
<td></td>
<td>(0.007)</td>
<td></td>
<td>(0.088)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>export dum&lt;sub&gt;−1&lt;/sub&gt;</td>
<td>-0.201***</td>
<td>-2.007***</td>
<td>-0.104***</td>
<td>-0.191***</td>
<td>-1.935***</td>
<td>-0.101***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>× log exports&lt;sub&gt;EE&lt;/sub&gt;</td>
<td>(0.021)</td>
<td>(0.259)</td>
<td>(0.016)</td>
<td>(0.019)</td>
<td>(0.253)</td>
<td>(0.016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mill. rev. dis&lt;sub&gt;−1&lt;/sub&gt;</td>
<td>0.001***</td>
<td>0.007***</td>
<td>0.0004***</td>
<td>0.001***</td>
<td>0.007***</td>
<td>0.0004***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>× log imports&lt;sub&gt;EE&lt;/sub&gt;</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-stat.</td>
<td>312.3</td>
<td>160.5</td>
<td>6,290.4</td>
<td>1,416.8</td>
<td>1,074.1</td>
<td>4674.5</td>
<td>1044.3</td>
<td>800.5</td>
</tr>
<tr>
<td>Observations</td>
<td>64,907</td>
<td>64,616</td>
<td>64,563</td>
<td>64,563</td>
<td>64,563</td>
<td>64,563</td>
<td>64,563</td>
<td>64,563</td>
</tr>
</tbody>
</table>

Note: The panel covers all industries and regions of Germany between 1996-2014. All regressions include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors in parentheses. Significance levels are indicated by * p < 0.1, ** p < 0.05, *** p < 0.01.

suggests using exports and imports at the industry level as instruments. However, since these industry aggregates themselves depend on the plants’ decision regarding the distinct number of occupation, they are likely not exogenous. Therefore, we follow the reasoning of Autor, Dorn and Hanson (2013) and use other high-income countries’ exports to and imports from China (CHN) as instruments for German exports and imports at the industry level. Regarding the sample of other high-income countries, we follow Dauth, Findeisen and Suedekum (2014) and use Australia, Canada, Japan, Norway, New Zealand, Sweden, Singapore, and the United Kingdom as instrument group. Since shipments to China affect exporters differently from non-exporters, we interact the log exports to China with a dummy capturing the export status of a plant in the previous business year. Furthermore, with three potentially endogenous regressors in Columns 2 and 3, it is not sufficient to specify just two instruments. Therefore, we add log exports to Eastern Europe (EE) interacted with a plant’s export status in the previous business year and the log of imports from Eastern Europe interacted with a plant’s millentile position in the revenue distribution from the previous business year as additional instruments, when necessary (and consider the same instrument group as for China). Accounting for exports to and imports from Eastern Europe as an additional set of instruments is motivated by the work of Dauth, Findeisen and Suedekum (2014), who show that trade exposure to China and trade exposure to Eastern Europe tend to have opposite consequences for the German economy.19

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19 As in their study, we associate Eastern Europe with Bulgaria, Czech Republic, Hungary, Poland, Romania, Slovakia, Slovenia, and the former USSR or its succession states Russian Federation, Belarus, Estonia, Latvia, Lithuania, Moldova, Ukraine, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, and Uzbekistan.
Column 4 reports the IV results when considering only log revenues as an explanatory variable (in addition to time, region, and industry fixed effects). In this case, we only need one instrument, for which we choose the interaction of log exports to China and the lagged exporter dummy. From the first stage regression in Table 5, we can conclude that this interaction term has indeed a positive and significant effect on log revenues, and the high F-test of excluded instruments shows that we need not be concerned about weak instrumentation. However, for this specification we cannot estimate a significant impact of log revenues on the log normalized number of tasks at stage two. Therefore, we try a different specification and add log imports from China as a second instrument at stage one. As we can see from Column 2 of Table 5, this second instrument itself does not exert a significant impact on log revenues. However, the F-statistic shows that our instruments are not weak and the overidentification test (Hansen’s J) does not reject validity of our instruments. More importantly, with the additional instrument, the estimated impact of log revenues on the log of normalized tasks at stage two becomes significant. In a further regression, we add the log number of distinct occupations and its interaction with log revenues as additional explanatory variables and instrument the now three endogenous regressors by the interaction of log exports to CHN with the lagged exporter dummy, the interaction of log exports to EE with the lagged exporter dummy, and log imports from EE with the lagged millentile position of a plant in the revenue distribution. From Columns 3-5 of Table 5, we see that the three instruments are significant in all three regressions. Also, the F-tests of excluded instruments are high in all three first-stage regressions.\textsuperscript{20} At the second stage, we find a (now larger) negative significant impact of log revenues on the log normalized number of tasks, whereas the coefficients of the log number of distinct occupations and its interaction with log revenues change their signs, when using an IV approach. In a final estimation, we add the log of imports as additional instrument. This allows us to test for overidentification and the high p-value reported in the Table 4 shows that we cannot reject validity of our instruments. Adding the additional instrument has only minor effects on the parameter estimates at stage two (see Column 7).\textsuperscript{21}

\textbf{Test of hypothesis 2:} To test for the sign of the correlation between wage differences inside a plant and its revenues, depending on $\lambda_0$, we proceed in two steps. In a first step, we estimate the relationship between within-plant wage differences, either measured by the coefficient variation of wages (Table 6)
<table>
<thead>
<tr>
<th>Dependent variable: log CV Daily wage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>OLS</td>
</tr>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>log revenues</td>
</tr>
<tr>
<td>(0.005)</td>
</tr>
<tr>
<td>log nr. of occupations</td>
</tr>
<tr>
<td>(0.083)</td>
</tr>
<tr>
<td>log revenues \times log nr. of occupations</td>
</tr>
<tr>
<td>(0.006)</td>
</tr>
<tr>
<td>Plant FE</td>
</tr>
<tr>
<td>Hansen J (p-val.)</td>
</tr>
<tr>
<td>R²</td>
</tr>
<tr>
<td>Adj. R²</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

| OLS                                  |
| (2)                                  |
| log revenues                         | 0.056*** |
| (0.013)                              |
| log nr. of occupations               | 1.425*** |
| (0.208)                              |
| log revenues \times log nr. of occupations | -0.075** |
| (0.014)                              |
| Plant FE                             | no       |
| Hansen J (p-val.)                    | n.a.     |
| R²                                   | 0.195    |
| Adj. R²                             | 0.195    |
| Observations                         | 126,483  |

| OLS                                  |
| (3)                                  |
| log revenues                         | 0.067*** |
| (0.021)                              |
| log nr. of occupations               | 0.118    |
| (2.111)                              |
| log revenues \times log nr. of occupations | 0.003    |
| (0.121)                              |
| Plant FE                             | yes      |
| Hansen J (p-val.)                    | n.a.     |
| R²                                   | 0.767    |
| Adj. R²                             | 0.688    |
| Observations                         | 126,483  |

| IV                                   |
| (4)                                  |
| log revenues                         | 0.129*** |
| (0.027)                              |
| log nr. of occupations               | 0.221    |
| (2.148)                              |
| log revenues \times log nr. of occupations | 0.001    |
| (0.123)                              |
| Plant FE                             | no       |
| Hansen J (p-val.)                    | 0.172    |
| R²                                   | 0.688    |
| Adj. R²                             | 0.688    |
| Observations                         | 64,905   |

| IV                                   |
| (5)                                  |
| log revenues                         | 0.127*** |
| (0.027)                              |
| log nr. of occupations               | 0.118    |
| (2.111)                              |
| log revenues \times log nr. of occupations | 0.003    |
| (0.121)                              |
| Plant FE                             | no       |
| Hansen J (p-val.)                    | 0.196    |
| R²                                   | 0.688    |
| Adj. R²                             | 0.688    |
| Observations                         | 64,614   |

| IV                                   |
| (6)                                  |
| log revenues                         | 0.038    |
| (0.067)                              |
| log nr. of occupations               | 0.118    |
| (2.111)                              |
| log revenues \times log nr. of occupations | 0.003    |
| (0.121)                              |
| Plant FE                             | no       |
| Hansen J (p-val.)                    | n.a.     |
| R²                                   | 0.172    |
| Adj. R²                             | 0.172    |
| Observations                         | 64,775   |

| IV                                   |
| (7)                                  |
| log revenues                         | 0.026    |
| (0.067)                              |
| log nr. of occupations               | 0.221    |
| (2.148)                              |
| log revenues \times log nr. of occupations | 0.001    |
| (0.123)                              |
| Plant FE                             | no       |
| Hansen J (p-val.)                    | n.a.     |
| R²                                   | 0.196    |
| Adj. R²                             | 0.196    |
| Observations                         | 64,561   |

Note: The panel covers all industries and regions of Germany between 1996-2014. All regressions include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors in parentheses. Significance levels are indicated by * p < 0.1, ** p < 0.05, *** p < 0.01.

or the standard deviation of residual wages (Table 7). We thereby use the same empirical specifications and the same instruments as in our estimation for the relationship between the log normalized number of tasks and the log revenues and therefore do not report the first-stage results for the IV approaches as they are identical to those reported in Table 5 – except for very small changes in the number of observations. The results in Tables 6 and 7 indicate a clear positive relationship between revenues and within-plant wage differences. In the baseline specification of Column 1, a 10 percent increase in revenues accounts for a one percentage increase in the coefficient of variation of wages within plants. This effect more than doubles, when considering the standard deviation of residual wages instead of the coefficient of variation of wages as a measure of within-plant wage differences. Comparing Columns 2 and 3 in Table 6, we see that the coefficient for the log number of distinct occupations changes its sign, when accounting for plant-level fixed effects. This is probably a consequence of not controlling for workforce heterogeneity when computing the coefficient of variation of wages at the plant level. The plant fixed effect seems to (partly) address this problem. This conjecture is supported by the observation that, when relying on the standard deviation of residual wages, the coefficient of the log number of occupations remains almost unaffected when including plant fixed effects (see Columns 2 and 3 in Table 7).

In Columns 4 and 5 we report estimation results when instrumenting log revenues as the single endogenous regressor. Test statistics are consistent with the hypothesis that the interaction of log exports to
Table 7: The impact of revenues on within-plant wage dispersion II

<table>
<thead>
<tr>
<th></th>
<th>Dependent variable: log StDev Residual daily wage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>OLS</td>
</tr>
<tr>
<td>log revenues</td>
<td>0.185***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
</tr>
<tr>
<td>log nr. of occupations</td>
<td>1.042***</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
</tr>
<tr>
<td>log revenues \times log nr. of occupations</td>
<td>-0.055***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
</tr>
<tr>
<td>Plant FE</td>
<td>no</td>
</tr>
<tr>
<td>Hansen J (p-val.)</td>
<td>n.a.</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.293</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.292</td>
</tr>
<tr>
<td>Observations</td>
<td>126,483</td>
</tr>
</tbody>
</table>

Note: The panel covers all industries and regions of Germany between 1996-2014. All regressions include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

China with the plant’s lagged exporter dummy and the log of imports from China provide valid instruments, and the positive impact of revenues on intra-plant wage variation remains robust to the change in estimation strategy. In Columns 6 and 7 we treat all three regressors—log revenues, the log number of occupations and the interaction term of these two variables—as endogenous variables and instrument them with the variables outlined in Table 5. This instrumentation is not successful when measuring intra-plant wage differences by the coefficient of variation of wages, because none of the three instrumented variables exerts a significant effect at the second stage in this case (see Table 6). Things are better, when relying on the standard deviation of residual wages as a measure of within-plant wage differences, because we can at least estimate significant coefficients for the log number of occupations and the interaction of log revenues and the log number of occupations (see Table 7). The missing significance of log revenues may indicate a problem of multicollinearity, when relying on IV estimation.

In a second step of our test of hypothesis 2, we check whether the positive relationship between revenues and within-plant wage differences reported in Tables 6 and 7 is in line with a positive estimate of $\lambda_0$. For this purpose, we can first combine eqs. (4) and (12) to solve for

$$\left[ \frac{\sqrt{4 + \pi (2 - \pi)}}{\text{cvw}} - \pi \right]_{t, \omega} = \lambda_0 \left( \frac{b}{z} \right)_{t, \omega},$$

(43)
where $t$ and $\omega$ are year and plant indices, respectively. Eq. (43) proposes an empirically testable link between $cvw$ and $b/z$. Following the reasoning in our model, we use the coefficient of variation of wages (and not the standard deviation of residual wages) for constructing the left-hand side variable in eq. (43), estimate the model with OLS, and report the results of this estimation in Columns 1-3 of Table 8. The estimate of $\lambda_0$ is positive in all specifications and, together with the estimation results on the relationship between the within-plant wage dispersion and revenues, we take this as strong supportive evidence for hypothesis 2.

For a further check of whether the structure of our theoretical model is meaningful, we can note that constant markup pricing establishes $\ell(\omega) = \left[\frac{(\sigma - 1) - \pi}{\sigma}\right] r(\omega) / w$ in our model. Substitution into eq. (17) gives $\ell(\omega) = \gamma[\lambda_0 + \pi(1 + \nu n(\omega))]$, which, in view of Eqs. (4) and (12), establishes a log-linear relationship between the coefficient of variation of wages within plants, the normalized number of tasks, and the number of production workers in this plant:

$$
\log \left[ \frac{cvw b}{z} \right]_{t,\omega} = \log \left[ \frac{\sqrt{4 + \pi(2 - \pi)}}{\gamma^{1/\gamma}} \right] - \frac{1}{\gamma} \ln \ell_{t,\omega}.
$$

We can construct the left-hand side variable and estimate eq. (44), using OLS. This gives the results in Columns 4-6 of Table 8, which support a positive level of $\gamma$ (associated with a negative coefficient), except for the case with plant fixed effects, in which the remaining variation in the data does not allow for estimating a significant coefficient for log employment. However, we take the results reported in Columns 4 and 5 as further supportive evidence for the structure of our model.

### 6 Concluding Remarks

We document empirically that workers in larger plants perform fewer tasks and that a dominant part of residual wage inequality materializes within plants, layers of hierarchies, and occupations.

Based on these observations, we augment the Melitz (2003) heterogeneous-firm model with a production technology for the internal labor market that allows plants to assign the range of tasks that need to be performed to an endogenous count of occupations. The within-plant wage dispersion changes with the plant’s choice of the count of occupations because workers in our model have a core ability that makes them most efficient at performing one specific task and monotonically less efficient in tasks that are farther from their core ability. A plant can therefore achieve a labor efficiency gain from narrowing the range of tasks performed by workers when raising the count of occupations to which it assigns tasks. More
Table 8: Estimation of the magnification parameter $\lambda_0$ and the semi-elasticity of fixed costs $\gamma$

<table>
<thead>
<tr>
<th></th>
<th>Dep. var.: $\sqrt{4 + \pi(2 - \pi)}/cvw - \pi$</th>
<th>Dep. var.: logarithm of $cvw \times b/z$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$b/z$</td>
<td>3.302**</td>
<td>4.258***</td>
</tr>
<tr>
<td></td>
<td>(0.820)</td>
<td>(1.136)</td>
</tr>
<tr>
<td>log employment</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>-0.996***</td>
<td>-1.813***</td>
</tr>
<tr>
<td></td>
<td>(-0.350)</td>
<td>(0.601)</td>
</tr>
<tr>
<td>Sector and Region FE</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Plant FE</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.001</td>
<td>0.007</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.001</td>
<td>0.006</td>
</tr>
<tr>
<td>Observations</td>
<td>126,483</td>
<td>126,483</td>
</tr>
</tbody>
</table>

Note: The panel covers all industries and regions of Germany between 1996-2014. All regressions include time fixed effects. Standard errors in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

specialized occupations allow the plant to narrow the range of tasks performed by workers within each job and, because workers’ core abilities are distributed over the task range, raise average labor efficiency. Extending the count of occupations, however, comes at an overhead cost because a larger number of occupations aggravates the span of control and generates coordination frictions that increase overhead costs. In equilibrium, inherently more productive plants and exporters adopt a larger count of occupations and thus boost their elemental productivity further with higher labor efficiency, by concentrating occupations on narrower task ranges. Under certain magnitudes of the baseline labor efficiency of workers, which we find confirmed in our estimation of the model, more productive plants end up with a wider dispersion of worker efficiencies. A worker’s wage is linked to the individual labor efficiency on the job, so that more productive plants exhibit higher wage dispersion in equilibrium.

For the open economy case, we show that trade between symmetric countries exerts opposing effects on exporters and non-exporters regarding their decisions on expanding or contracting the number of occupations. Exporters benefit from foreign market access, find it easier to cover their fixed costs, and therefore add new occupations to achieve higher labor efficiency by concentrating occupations on narrower task ranges. Non-exporters, in contrast, face stronger competition, due to the employment expansion of local exporters and the market entry of foreign firms, and therefore reduce the number of occupations—accepting lower labor efficiency in order to economize on fixed overhead costs. The asymmetric responses of exporting and non-exporting plants regarding the operation of occupations widens...
the gap in the within-plant dispersion of wages between plants and leads to an increase in economy-wide wage inequality, provided that larger plants experience higher wage dispersion prior to the globalization shock. As a consequence, gains from trade are accompanied by higher wage inequality in all open economies.

Combining matched plant–worker data from Germany with time-varying worker survey information on the tasks that workers perform on their jobs in Germany, we can test two main hypotheses from our theoretical model. First, we confirm a negative link between plant-level revenues and the number of tasks, using OLS. This result extends to a more robust empirical specification, in which we instrument plant-level revenues with sector-level trade shocks that mimic changes in German trade exposure with trade shocks between China or Eastern Europe on the one side and other high-income countries on the other side. Second, our model suggests a positive relationship between revenues and wage dispersion at the plant level under specific conditions. To test this hypothesis, we specify similar OLS and IV regressions as for the former hypothesis and establish a robust positive relationship between plant size and within-plant wage dispersion. Finally, we infer from our theoretical model structural relationships that allow us to test the required magnitudes of underlying parameters, including a crucial magnification parameter, and we find estimates that establish internal consistency.

In future drafts of this paper, we plan to conduct full-scale structural estimation to infer the relevant model parameters. We will then establish in simulations to what extent the internal labor markets at German plants responded to globalization shocks over time.
References


A Appendix

A.1 Derivation of $\int_{\psi^*}^{\infty} \psi(\varphi) \, dG(\varphi)$ in the closed economy

Using Eqs. (16), (17), and (22) we can write profits in the closed economy as follows

$$\psi(\varphi) = \frac{r(\varphi)}{\sigma} \frac{\gamma - \sigma + 1}{\gamma} - w f_0 = \frac{r(\varphi)}{r(\varphi^*)} w f_0 - w f_0. \quad (A.1)$$

Substituting $r(\varphi)/r(\varphi^*) = (\varphi/\varphi^*)^\xi$ from eq. (20), we can then compute

$$\int_{\psi^*}^{\infty} \psi(\varphi) \, dG(\varphi) = w f_0(\varphi^*)^\xi g \int_{\psi^*}^{\infty} \varphi^{\xi-g-1} \, d\varphi - w f_0 \int_{\psi^*}^{\infty} \varphi^{-g-1} \, d\varphi = (\varphi^*)^{-g} w f_0 \frac{\xi}{g - \xi}. \quad (A.2)$$

Accounting for $1 - G(\varphi^*) = (\varphi^*)^{-g}$ then gives the respective expression in the main text.

A.2 Derivation of eqs. (28) and (29)

The average employment of production workers per plant in the closed economy can be computed according to

$$\int_{\psi^*}^{\infty} \ell(\varphi) \frac{f \, \sigma dG(\varphi)}{1 - G(\varphi^*)} = \ell(\varphi^*)(\varphi^*)^\eta \int_{\psi^*}^{\infty} \varphi^{\xi-g-1} \, d\varphi = (\varphi^*)^\eta \frac{g}{g - \xi}, \quad (A.3)$$

according to eq. (21). The employment-share weighted average of the variances of wages at the plant-level is then given by eq. (28). Substitution of eq. (24) for $\text{var}(\varphi)$ and $\ell(\varphi) = (\varphi/\varphi^*)^\xi \ell(\varphi^*)$ from eq. (21) further establishes

$$\text{Var} = \text{var}(\varphi^*) \frac{g - \xi}{(b - \lambda_0)^2} \left\{ b^2 (\varphi^*)^\eta \int_{\varphi^*}^{\infty} \varphi^{\xi-g-1} \, d\varphi - 2 b \lambda_0 (\varphi^*)^\eta \int_{\varphi^*}^{\infty} \varphi^{\xi-g-1} \, d\varphi \right. \right.$$

$$\left. + \left. \lambda_0^2 (\varphi^*)^\eta \right] \int_{\varphi^*}^{\infty} \varphi^{\xi-g-1} \, d\varphi \right\}, \quad (A.4)$$

where $b \equiv (\xi f_0/\gamma)^{1/\gamma}$ has been used. Solving for the integral gives

$$\text{Var} = \frac{\text{var}(\varphi^*)}{[(\xi f_0/\gamma)^{1/\gamma} - \lambda_0]^2} \left[ \left( \frac{\xi f_0}{\gamma} \right)^{\frac{2}{\gamma}} - 2 \lambda_0 \left( \frac{\xi f_0}{\gamma} \right)^{\frac{1}{\gamma}} \frac{g - \xi}{g - \xi + \xi/\gamma} + \lambda_0^2 \frac{g - \xi}{g - \xi + 2\xi/\gamma} \right] \quad (A.5)$$

and finally eq. (29).

A.3 Derivation of $\int_{\psi^*}^{\infty} \psi_t(\varphi) \, dG(\varphi)$ in the open economy

Using eqs. (16), (17), and (22), we can write profits of non-exporters as follows $\psi_t^d(\varphi) = [r^d(\varphi)/r^d(\varphi^*)]f_0 - f_0$, where $w = 1$ because of our choice of numéraire. Total profits of exporters are given by

$$\psi_t^e(\varphi) = (1 + r^{1-\sigma}) \frac{\xi}{\sigma} \frac{r^d(\varphi)}{\gamma - \sigma + 1} f_0 - f_x = (1 + r^{1-\sigma}) \frac{\xi}{r^d(\varphi^*)} f_0 - f_0 - f_x, \quad (A.6)$$
Solving for the integrals yields
\[ \int_{\varphi^*}^{\infty} \psi_l(\varphi) \, dG(\varphi) = \int_{\varphi^*}^{\varphi_x} \psi_l^d(\varphi) \, dG(\varphi) + \int_{\varphi^*}^{\infty} \psi_l^e(\varphi) \, dG(\varphi) \]
\[ = f_0(\varphi^*)^{-g} \int_{\varphi^*}^{\varphi_x} \varphi^{\xi-g-1} \, d\varphi + (1 + \tau^{1-\sigma}) \int_{\varphi^*}^{\infty} \varphi^{\xi-g-1} \, d\varphi - f_0g \int_{\varphi^*}^{\infty} \varphi^{\xi-g-1} \, d\varphi \]

(A.7)

which in view of \( 1 - G(\varphi^*) = (\varphi^*)^{-g} \), \( \chi = (\varphi_x^*/\varphi^*)^{-g} \), and eq. (34) can be rewritten as eq. (35).

**A.4 Derivation of eqs. (41) and (42)**

First of all, the average employment of production workers per plant in the open economy can be computed according to

\[ \int_{\varphi^*}^{\infty} \ell_t(\varphi) \, \frac{dG(\varphi)}{1 - G(\varphi^*)} = \int_{\varphi^*}^{\varphi_x} \ell_t^d(\varphi) \, \frac{dG(\varphi)}{1 - G(\varphi^*)} + \int_{\varphi^*}^{\infty} \ell_t^e(\varphi) \, \frac{dG(\varphi)}{1 - G(\varphi^*)} \]
\[ = \frac{\ell_t^d(\varphi^*)}{\ell_t^e(\varphi^*)} \frac{g - \xi}{g - \xi} \left[ \int_{\varphi^*}^{\varphi_x} \varphi^{\xi-g-1} \, d\varphi + (1 + \tau^{1-\sigma}) \int_{\varphi^*}^{\infty} \varphi^{\xi-g-1} \, d\varphi \right] \]
\[ = \frac{\ell_t^d(\varphi^*)}{f_0} \left( 1 + \frac{\chi f_x}{f_0} \right), \quad (A.9) \]

where the second equality sign follows from eq. (32) and the third equality sign follows from eq. (34). The economy-share weighted average of the variances of wages at the plant-level can then be computed in analogy to the closed economy:

\[ Var = \frac{g - \xi}{gl_t^d(\varphi^*)} \left( 1 + \frac{\chi f_x}{f_0} \right)^{-1} \left[ \int_{\varphi^*}^{\varphi_x} \text{var}_d(\varphi) \ell_t^d(\varphi) \, \frac{dG(\varphi)}{1 - G(\varphi^*)} + \int_{\varphi^*}^{\infty} \text{var}_e(\varphi) \ell_t^e(\varphi) \, \frac{dG(\varphi)}{1 - G(\varphi^*)} \right]. \]

(A.10)
We look at the integrals on the right-side separately. Following the derivation steps of the closed economy and accounting for \( b = (\xi f_0 / \chi)^{1/\gamma} \), we compute

\[
\int_{\varphi^*}^\infty \text{var}_d(\varphi) \frac{dG(\varphi)}{1 - G(\varphi^*)} = \ell^d_t(\varphi^*) \frac{\text{var}_d(\varphi^*)}{(b - \lambda_0)^2} \int_{\varphi^*}^\infty \varphi^{\xi-g-1} d\varphi
\]

\[
-2b\lambda_0(\varphi^*)^{g+\xi-\xi} \int_{\varphi^*}^\infty \varphi^{\xi-g-\xi-1} d\varphi + \lambda_0^2(\varphi^*)^{g-\xi-\frac{2\xi}{\gamma}} \int_{\varphi^*}^\infty \varphi^{\xi-g-\frac{2\xi}{\gamma}-1} d\varphi
\]

\[
= \ell^d_t(\varphi^*) \frac{g}{(b - \lambda_0)^2 g - \xi} \left\{ \left[ b^2 + 2b\lambda_0 \frac{g - \xi}{g - \xi + \xi/\gamma} + \lambda_0^2 \frac{g - \xi + 2\xi/\gamma}{g - \xi + 2\xi/\gamma} \right] \left[ 1 - \left( \frac{\varphi^*}{\varphi^*} \right)^{\xi-g} \right] \right\}
\]

\[
+ 2b\lambda_0 \frac{g - \xi}{g - \xi + 2\xi/\gamma} \left( \frac{\varphi^*}{\varphi^*} \right)^{\xi-g} \left[ \left( \frac{\varphi^*}{\varphi^*} \right)^{-\frac{\xi}{\gamma}} - 1 \right]
\]

\[
- \lambda_0^2 \frac{g - \xi}{g - \xi + 2\xi/\gamma} \left( \frac{\varphi^*}{\varphi^*} \right)^{\xi-g} \left[ \left( \frac{\varphi^*}{\varphi^*} \right)^{-\frac{2\xi}{\gamma}} - 1 \right].
\] (A.11)

For the second integral, we obtain

\[
\int_{\varphi^*}^\infty \text{var}_e(\varphi) \frac{dG(\varphi)}{1 - G(\varphi^*)} = \ell^d_t(\varphi^*) \frac{\text{var}_d(\varphi^*)}{(b - \lambda_0)^2} \left( 1 + \tau^{1-\sigma} \frac{\xi}{\sigma-1} \right) \int_{\varphi^*}^\infty \varphi^{\xi-g-1} d\varphi
\]

\[
-2b\lambda_0(\varphi^*)^{g+\xi-\xi} \int_{\varphi^*}^\infty \varphi^{\xi-g-\xi-1} d\varphi + \lambda_0^2(\varphi^*)^{g-\xi-\frac{2\xi}{\gamma}} \int_{\varphi^*}^\infty \varphi^{\xi-g-\frac{2\xi}{\gamma}-1} d\varphi
\]

\[
= \ell^d_t(\varphi^*) \frac{g}{(b - \lambda_0)^2 g - \xi} \left\{ \left[ b^2 + 2b\lambda_0 \frac{g - \xi}{g - \xi + \xi/\gamma} + \lambda_0^2 \frac{g - \xi + 2\xi/\gamma}{g - \xi + 2\xi/\gamma} \right] \left( \frac{\varphi^*}{\varphi^*} \right)^{\xi-g}
\]

\[
+ 2b\lambda_0 \frac{g - \xi}{g - \xi + 2\xi/\gamma} \left( \frac{\varphi^*}{\varphi^*} \right)^{\xi-g} \left[ \left( \frac{\varphi^*}{\varphi^*} \right)^{-\frac{\xi}{\gamma}} - 1 \right]
\]

\[
- \lambda_0^2 \frac{g - \xi}{g - \xi + 2\xi/\gamma} \left( \frac{\varphi^*}{\varphi^*} \right)^{\xi-g} \left[ \left( \frac{\varphi^*}{\varphi^*} \right)^{-\frac{2\xi}{\gamma}} - 1 \right].
\] (A.12)

Substituting eqs. (A.11) and (A.12) into (A.9) and accounting for \( \varphi^*_x / \varphi^* = \chi^{-1/g} \) and eqs. (29) and (34), we arrive at eq. (41), with \( V(\chi) \) given by eq. (42). From \( \chi < 1 \) it follows that \( \chi^{\frac{\xi}{\gamma} - 1} > \chi^{\frac{2\xi}{\gamma} - 1} \). Noting further that \( \left( \frac{\xi f_0}{\chi} \right)^{1/\gamma} > \lambda_0 \), it follow from eq. (42) that

\[
2 \left[ 1 - \left( 1 + \chi^{\frac{\xi}{\gamma}} \frac{f_x}{f_0} \right)^{-\frac{\xi}{\gamma}} \chi^{\frac{\xi}{\gamma}} \right] > 1 - \left( 1 + \chi^{\frac{\xi}{\gamma}} \frac{f_x}{f_0} \right)^{-\frac{\xi}{\gamma}} \chi^{\frac{2\xi}{\gamma}}
\] (A.13)

or, equivalently,

\[
2 > \left[ 1 + \left( 1 + \chi^{\frac{\xi}{\gamma}} \frac{f_x}{f_0} \right)^{-\frac{\xi}{\gamma}} \chi^{\frac{\xi}{\gamma}} \right]
\] (A.14)

is sufficient for \( V(\chi) > 0 \). In view of \( [1 + \chi^{\xi/g} f_x / f_0]^{-1/\gamma} \chi^{\xi/(\gamma g)} < 1 \), this establishes the positive sign of \( V(\chi) \).

48
A.5 Marginal effect of a change in $\tau$ on $\text{Var}$

In the main text, we argue that the impact of a change in $\tau$ on the economy-wide variance of wages $\text{Var}$ is not necessarily monotonic. To show this result formally, we use the following definitions:

$$v(\chi) \equiv \frac{\chi^{1-\frac{\xi}{g}}}{1 + \chi f_x/f_o}, \quad \text{and} \quad \hat{V}(\chi) \equiv v(\chi)V(\chi). \quad (A.15)$$

From eq. (41), we can conclude that monotonicity of $\text{Var}$ in $\tau$ requires monotonicity $\hat{V}(\chi)$. Furthermore, it follows from $\hat{V}(0) = 0$ and $\hat{V}(\chi) > 0$ for all $\chi > 0$ (see Appendix A.4) that $V'(\chi) > 0$ at low levels of $\chi$. To see whether $V'(\chi)$ also extends to high levels of $\chi$, we can differentiate $\hat{V}(\chi)$. This gives

$$\hat{V}'(\chi) = v'(\chi)V(\chi) + v(\chi)V'(\chi), \quad (A.16)$$

with

$$v'(\chi) = \frac{v(\chi) 1 - (\xi/g)[1 + \chi f_x/f_o]}{\chi} \quad (A.17)$$

and

$$V'(\chi) \equiv 2 \left( \frac{\xi f_o}{\gamma} \right)^{\frac{1}{\gamma}} \frac{g - \xi + 2\xi/\gamma}{g - \xi + \xi/\gamma} \frac{1}{\chi} \left\{ \frac{\xi}{g} \chi^{\frac{\xi}{g}} \left[ 1 - \left( 1 + \chi^{\frac{\xi}{g}} f_x/f_o \right)^{-\frac{1}{\gamma}} \right] + \frac{\xi}{g} \frac{f_x}{f_0} \left[ 1 - \left( 1 + \chi^{\frac{\xi}{g}} f_x/f_o \right)^{-\frac{1}{\gamma}} \right] \right\}$$

- $\frac{\lambda_0}{\chi} \left\{ \frac{2\xi}{g} \chi^{\frac{\xi}{g}} \left[ 1 - \left( 1 + \chi^{\frac{\xi}{g}} f_x/f_o \right)^{-\frac{2}{\gamma}} \right] + \frac{\xi}{g} \chi^{\frac{\xi}{g}} \frac{f_x}{f_0} \left[ 1 - \left( 1 + \chi^{\frac{\xi}{g}} f_x/f_o \right)^{-\frac{2}{\gamma}} \right] \right\}$.

(A.18)

according to eq. (42). Rearranging terms, we get $\hat{V}'(\chi) = [v(\chi)/\chi] \left[ \hat{V}_0(\chi) - \hat{V}_1(\chi) \right]$, with

$$\hat{V}_0(\chi) \equiv 2 \left( \frac{\xi f_o}{\gamma} \right)^{\frac{1}{\gamma}} \frac{g - \xi + 2\xi/\gamma}{g - \xi + \xi/\gamma} \frac{1}{\chi} \left\{ \frac{\xi}{g} \chi^{\frac{\xi}{g}} \left[ 1 - \left( 1 + \chi^{\frac{\xi}{g}} f_x/f_o \right)^{-\frac{1}{\gamma}} \right] \right\} \left[ \frac{1}{1 + \chi f_x/f_o} + \frac{\xi}{g} - \frac{\xi}{g} \right]$$

+ $\frac{\xi}{g} \frac{f_x/f_0}{1 + \chi f_x/f_o} \left[ 1 - \left( 1 + \chi^{\frac{\xi}{g}} f_x/f_o \right)^{-\frac{1}{\gamma}} \right] \chi^{\frac{\xi}{g}}$.

(A.19)

and

$$\hat{V}_1(\chi) \equiv \frac{\lambda_0}{\chi} \left\{ \frac{\xi}{g} \chi^{\frac{\xi}{g}} \left[ 1 - \left( 1 + \chi^{\frac{\xi}{g}} f_x/f_o \right)^{-\frac{1}{\gamma}} \right] \right\} \left[ \frac{1}{1 + \chi f_x/f_o} + \frac{\xi}{g} - \frac{\xi}{g} \right]$$

+ $\frac{\xi}{g} \frac{f_x/f_0}{1 + \chi f_x/f_o} \left[ 1 - \left( 1 + \chi^{\frac{\xi}{g}} f_x/f_o \right)^{-\frac{1}{\gamma}} \right] \chi^{\frac{\xi}{g}}$.

(A.20)
Recollecting from eq. (22) that $n^* > 0$ requires $(\xi f_0/\gamma)^{1/\gamma} > \lambda_0$ and noting that $\epsilon_{hi} < 1$ establishes

$$
\begin{align*}
2g - \xi + 2\xi/\gamma \chi_x f_x/f_0 
&\quad \frac{1 - \left(1 + \chi_x f_x/f_0\right)^{-\frac{1}{\gamma}}}{1 + \chi_x f_x/f_0} \left[1 - \left(1 + \chi_x f_x/f_0\right)^{-\frac{1}{\gamma}}\right] - \frac{\chi_x f_x/f_0}{1 + \chi_x f_x/f_0} \left[1 - \left(1 + \chi_x f_x/f_0\right)^{-\frac{1}{\gamma}}\right] \\
\geq g - \xi + 2\xi/\gamma \chi_x f_x/f_0 \left[1 - \left(1 + \chi_x f_x/f_0\right)^{-\frac{1}{\gamma}}\right] - \frac{\chi_x f_x/f_0}{1 + \chi_x f_x/f_0} \left[1 - \left(1 + \chi_x f_x/f_0\right)^{-\frac{1}{\gamma}}\right]
\end{align*}
$$

we can safely conclude that

$$
\bar{V}(\chi) = 2g - \xi^2 + \xi/\gamma \left[1 - \frac{\xi}{g} + \frac{\xi}{g^2} \gamma\right] - \left[1 + \left(1 + \chi_x f_x/f_0\right)^{-\frac{1}{\gamma}}\right] \left[1 - \frac{\xi}{g} + \frac{\xi}{g^2} \gamma\right]
$$

$$
= g - \xi + 2\xi/\gamma \left[1 - \left(1 + \chi_x f_x/f_0\right)^{-\frac{1}{\gamma}}\right] - \frac{2\xi}{g^2} \chi_x f_x/f_0 \left[1 + \left(1 + \chi_x f_x/f_0\right)^{-\frac{1}{\gamma}}\right] \geq 0
$$

(A.21)

is sufficient for $\bar{V}'(\chi) < 0$. We can distinguish two cases regarding the ranking of

$$
g - \xi + 2\xi/\gamma \leq \frac{1 + \chi_x f_x/f_0}{\gamma}^{-\frac{1}{\gamma}}.
$$

(A.22)

If the left-hand side of (A.22) is strictly smaller than the right-hand side, $\bar{V}(\chi) > 0$ follows from the first line in eq. (A.21). If however, the left-hand side of (A.22) is strictly smaller than the right-hand side, $\bar{V}(\chi) > 0$ follows from the second line of eq. (A.21). This proves that $\bar{V}'(\chi) > 0$ holds for all possible realizations of $\chi$. 

50